Detection of the ISW effect and corresponding dark energy constraints

J. D. McEwen[1], P. Vielva[2,3], M. P. Hobson[1], E. Martinez-González[2], A. N. Lasenby[1], J. Delabrouille[3]

[1] Astrophysics Group
University of Cambridge

[2] Instituto de Física de Cantabria
Universidad de Cantabria

[3] Laboratoire APC
Collège de France

15 November 2005

Outline

Outline

ISW effect
Physical origin

- Photons blue (red) shifted when fall into (out of) potential wells
- Evolution of potential during photon propagation $\rightarrow$ net change in photon energy
- Large scale phenomenon
  (cosmic variance limited $\rightarrow$ require full-sky maps)
- Only present in non-flat universes or flat universes with dark energy

Temperature perturbation

$$\frac{\delta T}{T} = 2 \int \frac{\dot{\Phi}}{c^2} \frac{\mathrm{d}\ell}{c}$$

where $\mathrm{d}\ell$ is the element of proper distance. In Einstein de-Sitter universe (no Λ), $\Phi_k \sim \delta_k / a$ and linear growth law for $\Omega = 1$ is $\delta_k \sim a$. Thus $\dot{\Phi} \neq 0$ only when Ω diverges significantly from unity.

ISW effect
Physical origin

- Photons blue (red) shifted when fall into (out of) potential wells
- Evolution of potential during photon propagation
  $\rightarrow$ net change in photon energy
- Large scale phenomenon
  (cosmic variance limited $\rightarrow$ require full-sky maps)
- Only present in non-flat universes or flat universes with dark energy

Temperature perturbation

$$\frac{\delta T}{T} = 2 \int \frac{\dot{\Phi}}{c^2} \frac{\mathrm{d}\ell}{c}$$

where $\mathrm{d}\ell$ is the element of proper distance. In Einstein de-Sitter universe (no Λ), $\Phi_k \sim \delta_k / a$ and linear growth law for $\Omega = 1$ is $\delta_k \sim a$. Thus $\dot{\Phi} \neq 0$ only when Ω diverges significantly from unity.

ISW effect
Physical origin

- Photons blue (red) shifted when fall into (out of) potential wells
- Evolution of potential during photon propagation
  $\rightarrow$ net change in photon energy
- Large scale phenomenon
  (cosmic variance limited $\rightarrow$ require full-sky maps)
- Only present in non-flat universes or flat universes with dark energy

Temperature perturbation

$$\frac{\delta T}{T} = 2 \int \frac{\dot{\Phi}}{c^2} \frac{\mathrm{d}\ell}{c}$$

where $\mathrm{d}\ell$ is the element of proper distance. In Einstein de-Sitter universe (no Λ), $\Phi_k \sim \delta_k / a$ and linear growth law for $\Omega = 1$ is $\delta_k \sim a$. Thus $\dot{\Phi} \neq 0$ only when Ω diverges significantly from unity.

ISW effect
Physical origin

- Photons blue (red) shifted when fall into (out of) potential wells
- Evolution of potential during photon propagation $\rightarrow$ net change in photon energy
- Large scale phenomenon (cosmic variance limited $\rightarrow$ require full-sky maps)
- Only present in non-flat universes or flat universes with dark energy

Temperature perturbation

$$\frac{\delta T}{T} = 2 \int \frac{\dot{\Phi}}{c^2} \frac{\mathrm{d}\ell}{c}$$

where $\mathrm{d}\ell$ is the element of proper distance. In Einstein de-Sitter universe (no Λ), $\Phi_k \sim \delta_k / a$ and linear growth law for $\Omega = 1$ is $\delta_k \sim a$. Thus $\dot{\Phi} \neq 0$ only when Ω diverges significantly from unity.

Detecting the ISW effect

Cross-correlating the CMB with LSS

- Cannot directly separate the ISW signal from CMB anisotropies
- Detected by cross-correlating CMB anisotropies with tracers of large scale structure
  (first proposed by Crittenden & Turok 1996)
- Detections used to place constraints on dark energy
- Previous works
  - Real space angular correlation function
    (e.g. Boughn & Crittenden 2002)
  - Harmonic space cross-angular power spectrum
    (e.g. Afshordi et al. 2004)
  - Wavelet space covariance (Vielva et al. 2005)
- We extend spherical wavelet approach to directional wavelets (no reason to expect azimuthally symmetric structures)

Detecting the ISW effect

Cross-correlating the CMB with LSS

- Cannot directly separate the ISW signal from CMB anisotropies
- Detected by cross-correlating CMB anisotropies with tracers of large scale structure
  (first proposed by Crittenden & Turok 1996)
- Detections used to place constraints on dark energy
- Previous works
  - Real space angular correlation function
    (e.g. Boughn & Crittenden 2002)
  - Harmonic space cross-angular power spectrum
    (e.g. Afshordi et al. 2004)
  - Wavelet space covariance (Vielva et al. 2005)
- We extend spherical wavelet approach to directional wavelets (no reason to expect azimuthally symmetric structures)

Detecting the ISW effect
Cross-correlating the CMB with LSS

- Cannot directly separate the ISW signal from CMB anisotropies
- Detected by cross-correlating CMB anisotropies with tracers of large scale structure
  (first proposed by Crittenden & Turok 1996)
- Detections used to place constraints on dark energy
- Previous works
  - Real space angular correlation function
    (e.g. Boughn & Crittenden 2002)
  - Harmonic space cross-angular power spectrum
    (e.g. Afshordi et al. 2004)
  - Wavelet space covariance (Vielva et al. 2005)
- We extend spherical wavelet approach to directional wavelets (no reason to expect azimuthally symmetric structures)

Detecting the ISW effect

Cross-correlating the CMB with LSS

- Cannot directly separate the ISW signal from CMB anisotropies
- Detected by cross-correlating CMB anisotropies with tracers of large scale structure
  (first proposed by Crittenden & Turok 1996)
- Detections used to place constraints on dark energy
- Previous works
  - Real space angular correlation function
    (e.g. Boughn & Crittenden 2002)
  - Harmonic space cross-angular power spectrum
    (e.g. Afshordi et al. 2004)
  - Wavelet space covariance (Vielva et al. 2005)
- We extend spherical wavelet approach to directional wavelets (no reason to expect azimuthally symmetric structures)

Detecting the ISW effect
Cross-correlating the CMB with LSS

- Cannot directly separate the ISW signal from CMB anisotropies
- Detected by cross-correlating CMB anisotropies with tracers of large scale structure
  (first proposed by Crittenden & Turok 1996)
- Detections used to place constraints on dark energy
- Previous works
  - Real space angular correlation function
    (e.g. Boughn & Crittenden 2002)
  - Harmonic space cross-angular power spectrum
    (e.g. Afshordi et al. 2004)
  - Wavelet space covariance (Vielva et al. 2005)
- We extend spherical wavelet approach to directional wavelets (no reason to expect azimuthally symmetric structures)

Outline

Spherical wavelet transform
Anisotropic dilation on the sphere

- Spherical wavelet transform (Antoine and Vandergheynst 1998; Wiaux et al. 2005)
- Stereographic projection Π
- Anisotropic dilation on the sphere

$$\mathcal{D}(a,b) = \Pi^{-1}\, d(a,b)\, \Pi$$

$$[\mathcal{D}(a,b)s](\omega) = [\lambda(a,b,\theta,\phi)]^{1/2}\, s(\omega_{1/a,1/b})$$

where

$\omega_{a,b} = (\theta_{a,b}, \phi_{a,b})$,

$\tan(\theta_{a,b}/2) = \tan(\theta/2)\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}$

$\tan(\phi_{a,b}) = \frac{b}{a}\tan(\phi)$

Spherical wavelet transform
Anisotropic dilation on the sphere

- Spherical wavelet transform (Antoine and Vandergheynst 1998; Wiaux et al. 2005)
- Stereographic projection Π
- Anisotropic dilation on the sphere

$$\mathcal{D}(a, b) = \Pi^{-1}\, d(a, b)\, \Pi$$

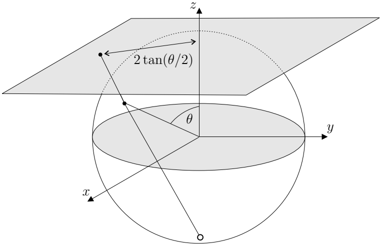
$$[\mathcal{D}(a, b)s](\omega) = [\lambda(a, b, \theta, \phi)]^{1/2}\, s(\omega_{1/a,1/b})$$

where

$\omega_{a,b} = (\theta_{a,b}, \phi_{a,b})$,

$\tan(\theta_{a,b}/2) = \tan(\theta/2)\sqrt{a^2\cos^2\phi + b^2\sin^2\phi}$

$\tan(\phi_{a,b}) = \frac{b}{a}\tan(\phi)$



Spherical wavelet transform
Anisotropic dilation on the sphere

- Spherical wavelet transform (Antoine and Vandergheynst 1998; Wiaux et al. 2005)
- Stereographic projection Π
- Anisotropic dilation on the sphere

$$\mathcal{D}(a, b) = \Pi^{-1}\, d(a, b)\, \Pi$$

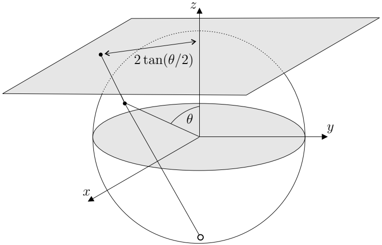
$$[\mathcal{D}(a, b)s](\omega) = [\lambda(a, b, \theta, \phi)]^{1/2}\, s(\omega_{1/a,1/b})$$

where

$$\omega_{a,b} = (\theta_{a,b}, \phi_{a,b}),$$

$$\tan(\theta_{a,b}/2) = \tan(\theta/2)\sqrt{a^2 \cos^2\phi + b^2 \sin^2\phi}$$

$$\tan(\phi_{a,b}) = \frac{b}{a}\tan(\phi)$$

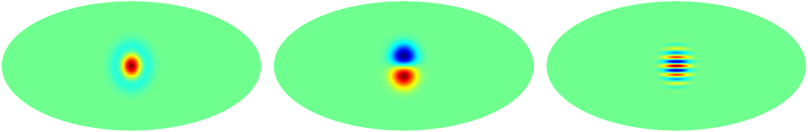


Spherical wavelet transform

Mother wavelets on the sphere

- Stereographic projection of admissible Euclidean mother wavelets

$$\psi(\omega) = [\Pi^{-1}\psi_{\mathbb{R}^2}](\omega)$$



(a) SMHW (b) SBW (c) SMW

Figure: Spherical wavelets at scale $a = b = 0.2$.

Spherical wavelet transform

- Motion on the sphere ($\equiv$ rotation)

$$[R(\rho)s](\omega) = s(\rho^{-1}\omega), \quad \rho \in \mathrm{SO}(3)$$

- Wavelet basis on the sphere

$$\{\psi_{a,b,\rho} \equiv R(\rho)\mathcal{D}(a,b)\psi, \ \rho \in \mathrm{SO}(3), a,b \in \mathbb{R}_*^+\}$$

- Spherical wavelet transform

$$W_\psi(a,b,\rho) \equiv \int_{S^2} \mathrm{d}\Omega(\omega)\, \psi_{a,b,\rho}^*(\omega)\, s(\omega)$$

- Fast algorithm (McEwen et al. 2005; Wandelt & Gorski 2001)

Spherical wavelet transform

- Motion on the sphere ($\equiv$ rotation)

$$[R(\rho)s](\omega) = s(\rho^{-1}\omega), \quad \rho \in \mathrm{SO}(3)$$

- Wavelet basis on the sphere

$$\{\psi_{a,b,\rho} \equiv R(\rho)\mathcal{D}(a,b)\psi, \ \rho \in \mathrm{SO}(3), \, a, b \in \mathbb{R}_*^+\}$$

- Spherical wavelet transform

$$W_\psi(a,b,\rho) \equiv \int_{S^2} \mathrm{d}\Omega(\omega) \, \psi^*_{a,b,\rho}(\omega) \, s(\omega)$$

- Fast algorithm (McEwen et al. 2005; Wandelt & Gorski 2001)

Spherical wavelet transform

- Motion on the sphere ($\equiv$ rotation)

$$[R(\rho)s](\omega) = s(\rho^{-1}\omega), \quad \rho \in \mathrm{SO}(3)$$

- Wavelet basis on the sphere

$$\{\psi_{a,b,\rho} \equiv R(\rho)\mathcal{D}(a,b)\psi, \ \rho \in \mathrm{SO}(3), \, a, b \in \mathbb{R}_*^+\}$$

- Spherical wavelet transform

$$W_\psi(a, b, \rho) \equiv \int_{S^2} \mathrm{d}\Omega(\omega) \, \psi^*_{a,b,\rho}(\omega) \, s(\omega)$$

- Fast algorithm (McEwen et al. 2005; Wandelt & Gorski 2001)

Spherical wavelet transform

- Motion on the sphere ($\equiv$ rotation)

$$[R(\rho)s](\omega) = s(\rho^{-1}\omega), \quad \rho \in \mathrm{SO}(3)$$

- Wavelet basis on the sphere

$$\{\psi_{a,b,\rho} \equiv R(\rho)\mathcal{D}(a,b)\psi, \ \rho \in \mathrm{SO}(3), \, a, b \in \mathbb{R}_*^+\}$$

- Spherical wavelet transform

$$W_\psi(a,b,\rho) \equiv \int_{S^2} \mathrm{d}\Omega(\omega) \, \psi_{a,b,\rho}^*(\omega) \, s(\omega)$$

- Fast algorithm (McEwen et al. 2005; Wandelt & Gorski 2001)

Outline

Wavelet covariance estimator

- Suitability of wavelets for detecting cross-correlations
- Wavelet covariance

$$\hat{X}_\psi^{\mathrm{NT}}(a,b,\gamma) = \frac{1}{N_{\alpha\beta}} \sum_{\alpha,\beta} \nu_{\alpha\beta} W_\psi^{\mathrm{N}}(a,b,\alpha,\beta,\gamma) W_\psi^{\mathrm{T}}(a,b,\alpha,\beta,\gamma)$$

- Average over orientations

$$\hat{X}_\psi^{\mathrm{NT}}(a,b) = \frac{1}{N_\gamma} \sum_{\gamma} \hat{X}_\psi^{\mathrm{NT}}(a,b,\gamma)$$

- Theoretical wavelet covariance

$$X_\psi^{\mathrm{NT}}(a,b,\gamma) = \sum_{\ell=0}^{\infty} {p_\ell}^2 \, b_\ell^{\mathrm{N}} \, b_\ell^{\mathrm{T}} \, C_\ell^{\mathrm{NT}} \sum_{m=-\ell}^{\ell} \left|(\psi_{a,b})_{\ell m}\right|^2$$

Wavelet covariance estimator

- Suitability of wavelets for detecting cross-correlations
- Wavelet covariance

$$\hat{X}_\psi^{\mathrm{NT}}(a,b,\gamma) = \frac{1}{N_{\alpha\beta}} \sum_{\alpha,\beta} \nu_{\alpha\beta} W_\psi^{\mathrm{N}}(a,b,\alpha,\beta,\gamma) W_\psi^{\mathrm{T}}(a,b,\alpha,\beta,\gamma)$$

- Average over orientations

$$\hat{X}_\psi^{\mathrm{NT}}(a,b) = \frac{1}{N_\gamma} \sum_{\gamma} \hat{X}_\psi^{\mathrm{NT}}(a,b,\gamma)$$

- Theoretical wavelet covariance

$$X_\psi^{\mathrm{NT}}(a,b,\gamma) = \sum_{\ell=0}^{\infty} {p_\ell}^2 \, b_\ell^{\mathrm{N}} \, b_\ell^{\mathrm{T}} \, C_\ell^{\mathrm{NT}} \sum_{m=-\ell}^{\ell} \left| (\psi_{a,b})_{\ell m} \right|^2$$

Wavelet covariance estimator

- Suitability of wavelets for detecting cross-correlations
- Wavelet covariance

$$\hat{X}_{\psi}^{\rm NT}(a,b,\gamma) = \frac{1}{N_{\alpha\beta}} \sum_{\alpha,\beta} \nu_{\alpha\beta} W_{\psi}^{\rm N}(a,b,\alpha,\beta,\gamma) W_{\psi}^{\rm T}(a,b,\alpha,\beta,\gamma)$$

- Average over orientations

$$\hat{X}_{\psi}^{\rm NT}(a,b) = \frac{1}{N_{\gamma}} \sum_{\gamma} \hat{X}_{\psi}^{\rm NT}(a,b,\gamma)$$

- Theoretical wavelet covariance

$$X_{\psi}^{\rm NT}(a,b,\gamma) = \sum_{\ell=0}^{\infty} p_{\ell}^{2}\, b_{\ell}^{\rm N}\, b_{\ell}^{\rm T}\, C_{\ell}^{\rm NT} \sum_{m=-\ell}^{\ell} \left|(\psi_{a,b})_{\ell m}\right|^{2}$$

Wavelet covariance estimator

- Suitability of wavelets for detecting cross-correlations
- Wavelet covariance

$$\hat{X}_{\psi}^{\mathrm{NT}}(a,b,\gamma) = \frac{1}{N_{\alpha\beta}} \sum_{\alpha,\beta} \nu_{\alpha\beta} W_{\psi}^{\mathrm{N}}(a,b,\alpha,\beta,\gamma) W_{\psi}^{\mathrm{T}}(a,b,\alpha,\beta,\gamma)$$

- Average over orientations

$$\hat{X}_{\psi}^{\mathrm{NT}}(a,b) = \frac{1}{N_{\gamma}} \sum_{\gamma} \hat{X}_{\psi}^{\mathrm{NT}}(a,b,\gamma)$$

- Theoretical wavelet covariance

$$X_{\psi}^{\mathrm{NT}}(a,b,\gamma) = \sum_{\ell=0}^{\infty} {p_{\ell}}^{2}\, b_{\ell}^{\mathrm{N}}\, b_{\ell}^{\mathrm{T}}\, C_{\ell}^{\mathrm{NT}} \sum_{m=-\ell}^{\ell} \left|(\psi_{a,b})_{\ell m}\right|^{2}$$

Comparison of wavelets

- Compare predicted signal-to-noise ratio

$$\mathrm{SNR}_\psi(a,b) = \frac{\left\langle \hat{X}_\psi^{\mathrm{NT}}(a,b) \right\rangle}{\Delta \hat{X}_\psi^{\mathrm{NT}}(a,b)}$$

where

$$\left[\Delta \hat{X}_\psi^{\mathrm{NT}}(a,b)\right]^2 = \sum_{\ell=0}^{\infty} \frac{1}{2\ell+1} \, {p_\ell}^4 \, (b_\ell^{\mathrm{N}})^2 \, (b_\ell^{\mathrm{T}})^2 \left(\sum_{m=-\ell}^{\ell} |(\psi_{a,b})_{\ell m}|^2 \right)^2 \left((C_\ell^{\mathrm{NT}})^2 + C_\ell^{\mathrm{TT}} C_\ell^{\mathrm{NN}} \right)$$

- Similar technique used to compare real, harmonic and wavelet space techniques for detection of cross-correlations
  $\rightarrow$ wavelets optimal on certain scales (Vielva et al. 2005)

Comparison of wavelets

- Compare predicted signal-to-noise ratio

$$\mathrm{SNR}_\psi(a,b) = \frac{\left\langle \hat{X}_\psi^{\mathrm{NT}}(a,b) \right\rangle}{\Delta \hat{X}_\psi^{\mathrm{NT}}(a,b)}$$

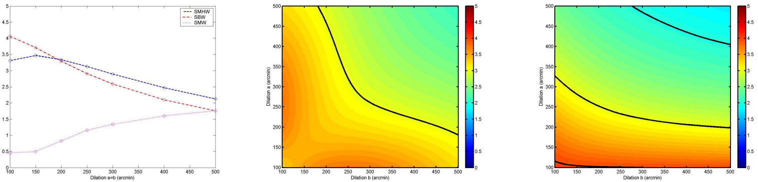
where

$$\left[\Delta \hat{X}_\psi^{\mathrm{NT}}(a,b)\right]^2 = \sum_{\ell=0}^{\infty} \frac{1}{2\ell+1} \, {p_\ell}^4 \, (b_\ell^{\mathrm{N}})^2 \, (b_\ell^{\mathrm{T}})^2 \left(\sum_{m=-\ell}^{\ell} |(\psi_{a,b})_{\ell m}|^2 \right)^2 \left((C_\ell^{\mathrm{NT}})^2 + C_\ell^{\mathrm{TT}} C_\ell^{\mathrm{NN}} \right)$$

- Similar technique used to compare real, harmonic and wavelet space techniques for detection of cross-correlations
  $\rightarrow$ wavelets optimal on certain scales (Vielva et al. 2005)

Comparison of wavelets

SNR plots

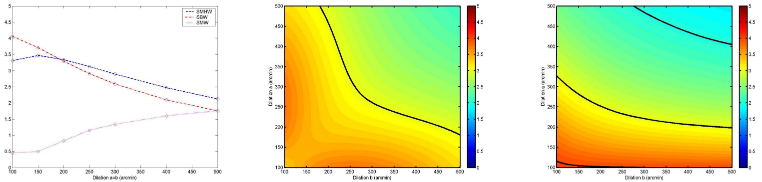


(a) All wavelets (b) SMHW (c) SBW

Figure: Expected SNR of the wavelet covariance estimator of CMB and radio source maps

- Don't consider SMW further
  (actually considered; as expected not effective)

Comparison of wavelets
SNR plots



(a) All wavelets (b) SMHW (c) SBW

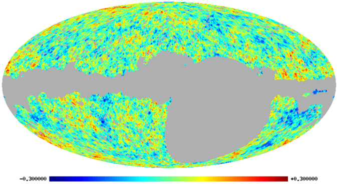
Figure: Expected SNR of the wavelet covariance estimator of CMB and radio source maps

- Don't consider SMW further
  (actually considered; as expected not effective)

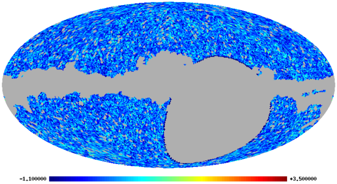
Outline

Analysis procedure

- Data



WMAP

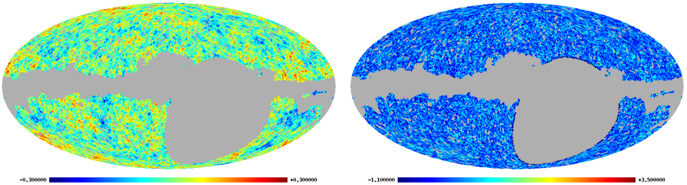


NVSS

- Analysis (scales; masks)
- Simulations
- Constraints on dark energy parameters

Analysis procedure

- Data

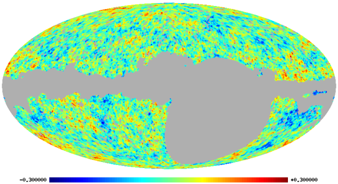


WMAP NVSS

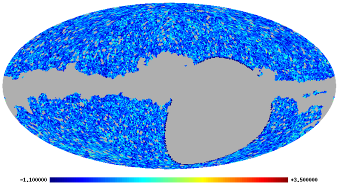
- Analysis (scales; masks)
- Simulations
- Constraints on dark energy parameters

Analysis procedure

- Data



WMAP

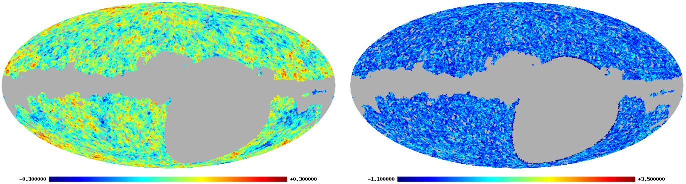


NVSS

- Analysis (scales; masks)
- Simulations
- Constraints on dark energy parameters

Analysis procedure

- Data



WMAP NVSS

- Analysis (scales; masks)
- Simulations
- Constraints on dark energy parameters

Outline

Scales and detections

- Scales

Scale	1	2	3	4	5	6	7
Dilation a	$100'$	$150'$	$200'$	$250'$	$300'$	$400'$	$500'$
Size on sky 1	$282'$	$424'$	$565'$	$706'$	$847'$	$1130'$	$1410'$
Size on sky 2	$31.4'$	$47.1'$	$62.8'$	$78.5'$	$94.2'$	$126'$	$157'$

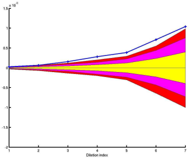
- Wavelet covariance plots

Scales and detections

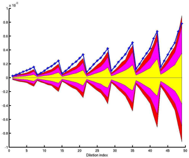
- Scales

Scale	1	2	3	4	5	6	7
Dilation a	$100'$	$150'$	$200'$	$250'$	$300'$	$400'$	$500'$
Size on sky 1	$282'$	$424'$	$565'$	$706'$	$847'$	$1130'$	$1410'$
Size on sky 2	$31.4'$	$47.1'$	$62.8'$	$78.5'$	$94.2'$	$126'$	$157'$

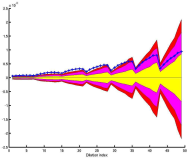
- Wavelet covariance plots



SMHW



SMHW



SBW

Significance of detections

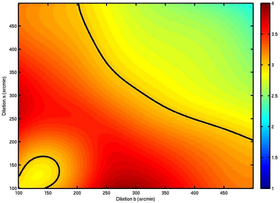
- Most significant detections
  - Wavelet covariance statistics appear Gaussian
    $\rightarrow N_\sigma$ direct indication of significance of detections
  - symmetric SMHW: 3.6σ; elliptical SMHW: 3.9σ; SBW: 3.9σ
- N_σ plots (2 and 3σ contours)

Significance of detections

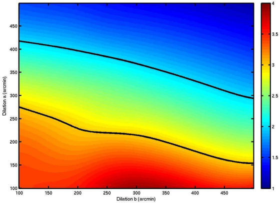
- Most significant detections
  - Wavelet covariance statistics appear Gaussian
    $\rightarrow N_\sigma$ direct indication of significance of detections
  - symmetric SMHW: 3.6σ; elliptical SMHW: 3.9σ; SBW: 3.9σ
- N_σ plots (2 and 3σ contours)

Significance of detections

- Most significant detections
  - Wavelet covariance statistics appear Gaussian
    $\rightarrow N_\sigma$ direct indication of significance of detections
  - symmetric SMHW: 3.6σ; elliptical SMHW: 3.9σ; SBW: 3.9σ
- N_σ plots (2 and 3σ contours)



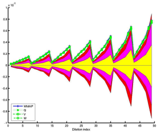
SMHW



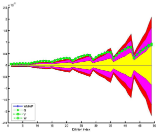
SBW

Systematics and foregrounds

- Systematics: individual WMAP receiver maps
  → systematics not likely source of detection
- Foregrounds: foreground dominated difference maps
  → foregrounds not likely source of detection



SMHW



SBW

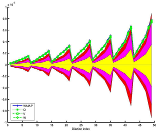
Individual receiver maps

SMHW SBW

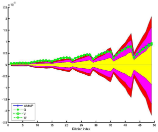
Difference maps

Systematics and foregrounds

- Systematics: individual WMAP receiver maps
  $\rightarrow$ systematics not likely source of detection
- Foregrounds: foreground dominated difference maps
  $\rightarrow$ foregrounds not likely source of detection

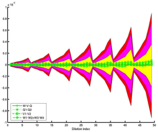


SMHW

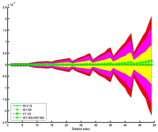


SBW

Individual receiver maps



SMHW



SBW

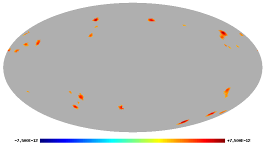
Difference maps

Localised regions
Detection

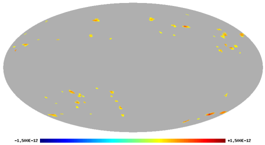
- Wavelets inherently provide spatial localisation (in addition to scale localisation)
- Threshold wavelet coefficient product maps to localise most likely sources

Localised regions
Detection

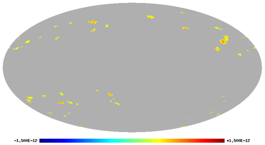
- Wavelets inherently provide spatial localisation (in addition to scale localisation)
- Threshold wavelet coefficient product maps to localise most likely sources



Symmetric SMHW



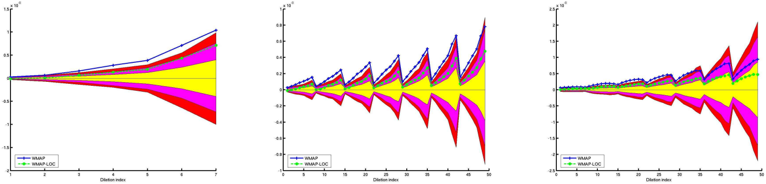
Elliptical SMHW



SBW

Localised regions
Removal

- Remove localised regions → ISW detection remains
  (Agrees with findings of Boughn and Crittenden 2004)



Symmetric SMHW | Elliptical SMHW | SBW

Dark energy constraints

- Compute theoretical wavelet covariance for range of models (w, Ω_Λ)

  (assume concordance model for other parameters; bias $b = 1.6$)

- Compare theoretical predictions with observations

$$\chi^2(w, \Omega_\Lambda) = \Delta^{\mathrm{T}} C^{-1} \Delta$$

  where

$$\Delta = [\hat{X}_\psi^{\mathrm{NT}}(a, b, \gamma) - X_\psi^{\mathrm{NT}}(a, b, \gamma | w, \Omega_\Lambda)]$$

- Compute likelihood

$$\mathcal{L}(w, \Omega_\Lambda) \propto \exp[-\chi^2(w, \Omega_\Lambda)/2]$$

Dark energy constraints

- Compute theoretical wavelet covariance for range of models (w, Ω_Λ)
  (assume concordance model for other parameters; bias $b = 1.6$)
- Compare theoretical predictions with observations

$$\chi^2(w, \Omega_\Lambda) = \Delta^{\mathrm{T}} C^{-1} \Delta$$

where

$$\Delta = [\hat{X}_\psi^{\mathrm{NT}}(a, b, \gamma) - X_\psi^{\mathrm{NT}}(a, b, \gamma | w, \Omega_\Lambda)]$$

- Compute likelihood

$$\mathcal{L}(w, \Omega_\Lambda) \propto \exp[-\chi^2(w, \Omega_\Lambda)/2]$$

Dark energy constraints

- Compute theoretical wavelet covariance for range of models (w, Ω_Λ)
  (assume concordance model for other parameters; bias $b = 1.6$)
- Compare theoretical predictions with observations

$$\chi^2(w, \Omega_\Lambda) = \Delta^{\mathrm{T}} C^{-1} \Delta$$

where

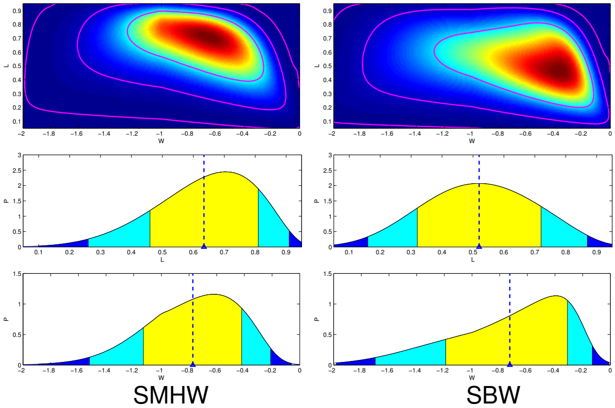
$$\Delta = [\hat{X}_\psi^{\mathrm{NT}}(a, b, \gamma) - X_\psi^{\mathrm{NT}}(a, b, \gamma | w, \Omega_\Lambda)]$$

- Compute likelihood

$$\mathcal{L}(w, \Omega_\Lambda) \propto \exp[-\chi^2(w, \Omega_\Lambda)/2]$$

Dark energy constraints

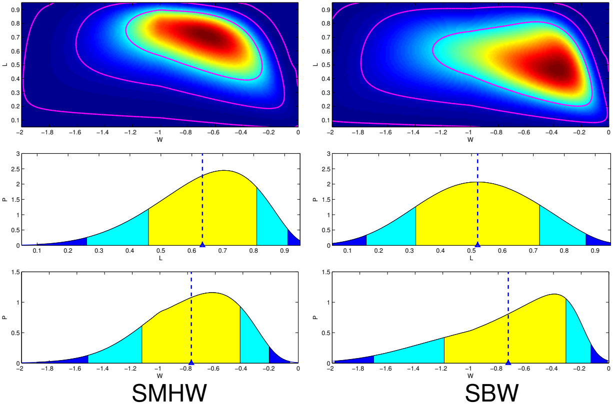
Likelihood surfaces (preliminary)



- Parameter estimates from mean of marginalised distributions
  - $\Omega_\Lambda = 0.63^{+0.18}_{-0.17}$; $w = -0.77^{+0.35}_{-0.36}$ using SMHW
  - $\Omega_\Lambda = 0.52^{+0.20}_{-0.20}$; $w = -0.73^{+0.42}_{-0.46}$ using SBW

Dark energy constraints

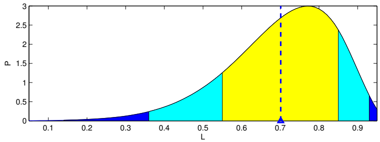
Likelihood surfaces (preliminary)

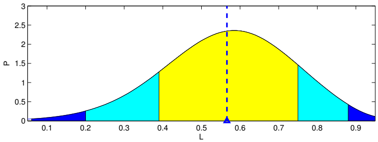


- Parameter estimates from mean of marginalised distributions
  - $\Omega_\Lambda = 0.63^{+0.18}_{-0.17}$; $w = -0.77^{+0.35}_{-0.36}$ using SMHW
  - $\Omega_\Lambda = 0.52^{+0.20}_{-0.20}$; $w = -0.73^{+0.42}_{-0.46}$ using SBW

Dark energy constraints

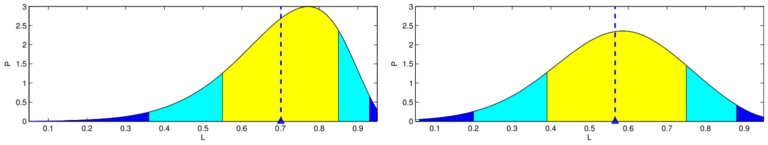
Parameter estimates (preliminary)





- Also considered case $w = -1$
  - $\Omega_\Lambda = 0.70^{+0.15}_{-0.15}$ using SMHW
  - $\Omega_\Lambda = 0.57^{+0.18}_{-0.18}$ using SBW
- Reject $\Omega_\Lambda = 0$ at $> 99\%$ significance
  - $\Omega_\Lambda > 0.1$ at 99.9% using SMHW
  - $\Omega_\Lambda > 0.1$ at 99.7% using SBW

Dark energy constraints
Parameter estimates (preliminary)



- Also considered case $w = -1$
  - $\Omega_\Lambda = 0.70^{+0.15}_{-0.15}$ using SMHW
  - $\Omega_\Lambda = 0.57^{+0.18}_{-0.18}$ using SBW
- Reject $\Omega_\Lambda = 0$ at $> 99\%$ significance
  - $\Omega_\Lambda > 0.1$ at 99.9% using SMHW
  - $\Omega_\Lambda > 0.1$ at 99.7% using SBW

Outline

Summary

- Using spherical wavelets to detect ISW effect
- Detection of ISW effect made at almost 4σ
- Independent evidence of dark energy
- Constrain dark energy

Summary

- Using spherical wavelets to detect ISW effect
- Detection of ISW effect made at almost 4σ
- Independent evidence of dark energy
- Constrain dark energy

Summary

- Using spherical wavelets to detect ISW effect
- Detection of ISW effect made at almost 4σ
- Independent evidence of dark energy
- Constrain dark energy