fast directional spherical wavelets

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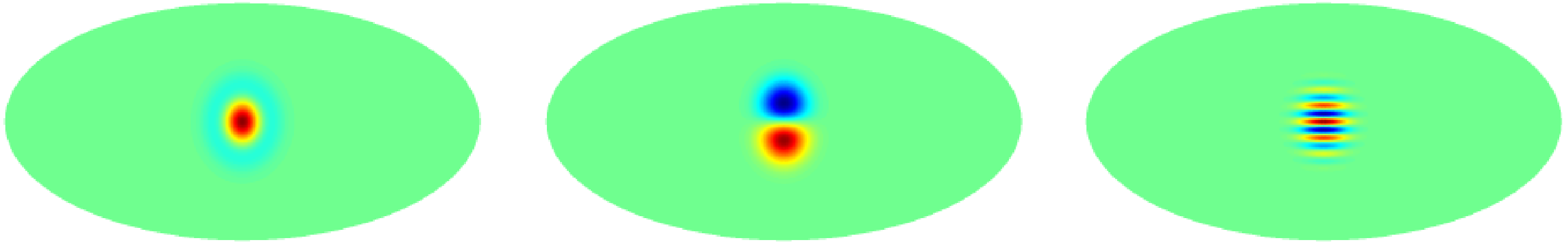
Overview

- Spherical wavelet analysis
- Fast algorithms
- Complexity
- Typical CPU time
- Problematics
- No preliminary results yet

Spherical wavelet analysis

- Ability to probe different scales, positions and – with directional wavelets – orientations
- Retain localisation information
- Full sky analysis → Wavelet analysis on the sphere
- Spherical wavelets
  - Azimuthally symmetric
  - Directional wavelets
  - Steerable wavelets (Wiaux et al. 2005a)



(a) Spherical Mexican hat wavelet (b) Spherical butterfly wavelet (c) Spherical real Morlet wavelet

Spherical wavelet analysis

- Non-Gaussianity analysis
  - CSWT linear: Gaussian sky → Gaussian coefficients
  - Examine skewness and kurtosis of wavelet coefficients
  - Previous detections of non-Gaussianity in WMAP with symmetric and directional spherical wavelets

    (Vielva et al. 2004 and McEwen et al. 2005 respectively)

Fast algorithms

- Azimuthally symmetric spherical wavelets

$$(\widehat{W_\psi^s})_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}}\, \widehat{\psi}_{\ell 0}^{*}\, \widehat{s}_{\ell m}$$

- Directional spherical wavelets

$$W_\psi^s(\alpha,\beta,\gamma) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} \sum_{m'=-\ell}^{\ell} \left[D_{mm'}^{\ell}(\alpha,\beta,\gamma)\, \widehat{\psi}_{\ell m'} \right]^{*} \widehat{s}_{\ell m}$$

  - Factor rotations (McEwen et al. 2005b, Wandelt & Gorski 2001)
  - Separation of variables (Wiaux et al. 2005b)

- Steerable wavelets (Wiaux et al. 2005a)

$$\left[R^{\hat{z}}(\chi)\Psi\right](\omega) = \sum_{m=1}^{M} k_m(\chi)\, \Psi_m(\omega) .$$

Complexity

- Resolution $L \sim l_{max} \sim \sqrt{N_{pix}}$
- Azimuthal resolution M (typically $M \ll L$)

- Asymptotic complexity:
  - Naïve: $\mathcal{O}(L^4 M)$
  - Azimuthally symmetric: $\mathcal{O}(L^3)$
  - Directional: $\mathcal{O}(L^3 M)$
  - Directional and steerable: $\mathcal{O}(L^3)$ (Infinite azimuthal resolution)

CPU time

- Typical execution time

TABLE II

TYPICAL EXECUTION TIME (MINUTES:SECONDS) FOR EACH ALGORITHM RUN ON AN INTEL P4-M 3GHZ LAPTOP WITH 512MB OF MEMORY.

Resolution		Algorithm execution time		
N_{side}	N_{pix}	Direct	Fast isotropic	Fast anisotropic
32	12,288	3:25:37	0:00.06	0:00.10
64	49,152	54:31.75	0:00.38	0:00.74
256	786,432	–	0:28.00	0:52.55
512	3,257,292	–	3:43.69	7:57.75
1024	12,582,912	–	28:23.85	71:31.68

Problematics

- CPU requirements
  - $\sim$1 CPU hr per scale
  - 5 scales, 3 orientations $\rightarrow$ $\sim$10 CPU hours per analysis
  - 300 Gaussian simulations + 300 non-Gaussian simulations
    $\rightarrow$ $\sim$6000 CPU hours (10 processors $\rightarrow$ 600 hours$\sim$25 days)
- Storage
  - $\sim$500MB per wavelet coefficient file (5 scales, 3 orientations)
  - $\rightarrow$ compute summary statistics, then delete wavelet coefficients
- Pre-processing
  - Generate extended coefficient masks requires wavelet transforms plus morphological operations (latter is extremely slow) $\rightarrow$ require simpler techniques for generating extended mask

Consolation:

- Linear $\rightarrow$ can do cases with foregrounds for minimal extra effort

Summary

- Powerful technique to probe for non-Gaussianity due to scale and spatial localisation – but may not be optimal for f_{NL} type non-Gaussianity
- Computationally intensive, especially directional analysis
- Large storage requirements
- Preliminary results forthcoming