A 6σ detection of non-Gaussianity in the WMAP 1-year data using directional spherical wavelets

astro-ph/0406604

Jason McEwen, Mike Hobson,
Anthony Lasenby & Daniel Mortlock

Cavendish Laboratory, Cambridge

Overview

- *Fast directional continuous spherical wavelet transform*
- Detection of *non-Gaussianity* in WMAP anisotropies using *directional spherical wavelet analysis*
  - If original map is Gaussian, then *wavelet coefficients also Gaussian*
  - *Skewness and kurtosis* of wavelet coefficients
  - Compare with *Monte Carlo simulations* to determine significance
- Conclusions and future work

CSWT

- Wavelets and non-Gaussianity
  - Ability to probe *different scales*
  - *Localise* non-Gaussian components

  $\Rightarrow$ Wavelet analysis ideal
- *Directional continuous spherical wavelet transform* (CSWT)

  (Antoine & Vandergheynst 1998)
- *Fast* directional CSWT (based on fast spherical convolution proposed by Wandelt and Gorski 2001)

Algorithm	Complexity
Direct	$\mathcal{O}(L^4 N_\gamma)$
Semi-fast	$\mathcal{O}(L^3 \log_2(L) N_\gamma)$
Fast	$\mathcal{O}(L^3 N_\gamma)$

$\Rightarrow$ Saving: $\mathcal{O}(L) \sim \mathcal{O}(\sqrt{N})$

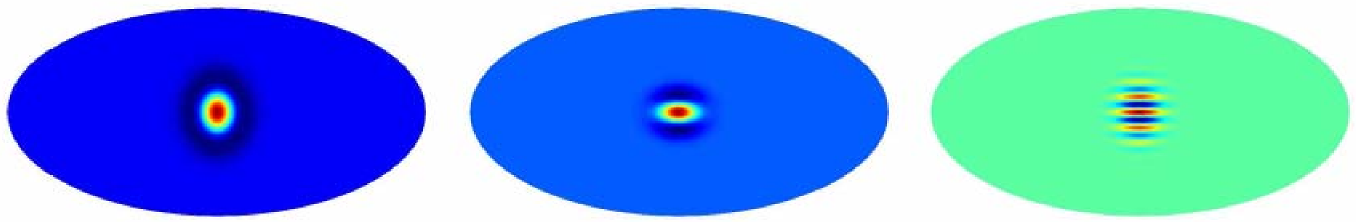
$\mathbf{N_{side}}$	Execution time (min:sec)		
	Direct	Semi-fast	Fast
8	00:01.19	00:01.12	00:00.01
16	00:18.60	00:17.38	00:00.04
32	05:01.48	04:43.06	00:00.21
256	-	-	01:54.15

Sun Fire 280R Server, Dual UltraSPARC III 900MHz Processors, 4GB Memory

Mother spherical wavelets

- Project directional Euclidean planar wavelets onto the sphere
- Directional extension of Mexican hat wavelet

  $\Rightarrow$ *elliptical* Mexican hat wavelets

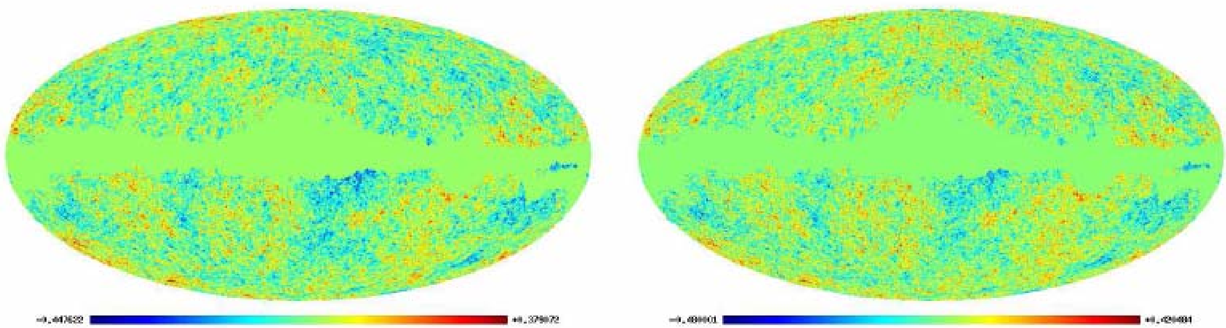


(a) Mexican hat $\epsilon = 0.00$ (b) Mexican hat $\epsilon = 0.95$ (c) Real Morlet $\boldsymbol{k} = (10, 0)^T$

Figure 2. Spherical wavelets (dilation $a = 750'$; size on sky $\xi = 2105'$).

Maps considered

- Co-added *WMAP map* based on construction described by Komatsu et al. (2003) (foreground removal described by Bennet et al. (2003))
- *Tegmark et al.* (2003) foreground removed map
  - Linear combination of bands, however weights also vary with scale



(a) Co-added WMAP team map

(b) Tegmark map

Figure 3. Preprocessed WMAP maps considered in the non-Gaussianity analysis.

Wavelet analysis

- Follow similar strategy to Vielva et al. (2003), extended to directional analysis
- Take *CSWT of maps* at a range of scales and, for direction wavelets, orientations

  (we consider 5 uniformly sampled orientations in domain $(0,\pi]$)

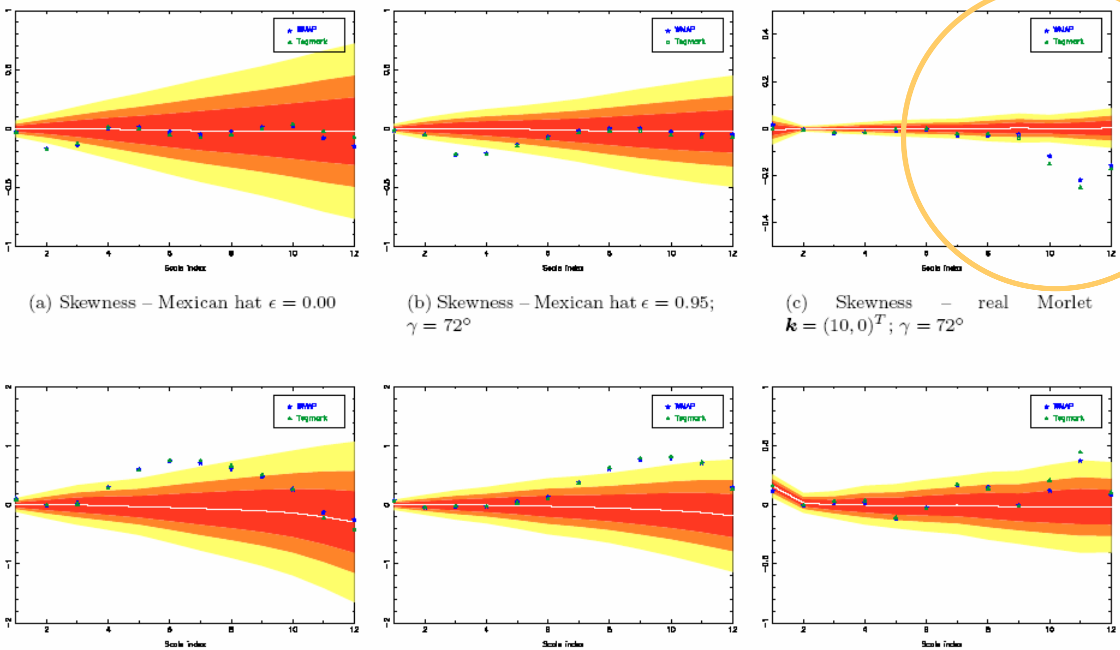
Table 1. Wavelet scales considered in the non-Gaussianity analysis. The effective size on the sky for a given scale are the same for both the elliptical Mexican hat and real Morlet wavelets.

Scale	1	2	3	4	5	6	7	8	9	10	11	12
Dilation a	$50'$	$100'$	$150'$	$200'$	$250'$	$300'$	$350'$	$400'$	$450'$	$500'$	$550'$	$600'$
Size on sky ξ	$141'$	$282'$	$424'$	$565'$	$706'$	$847'$	$988'$	$1129'$	$1269'$	$1409'$	$1549'$	$1689'$

- Construct and apply extended *coefficient exclusion masks*
- Consider *skewness and kurtosis of wavelet coefficients* to detect deviations from Gaussianity
- *Monte Carlo simulations*

Wavelet coefficient statistics

- Compare wavelet coefficient statistics with Monte Carlo simulations



(a) Skewness – Mexican hat $\epsilon = 0.00$

(b) Skewness – Mexican hat $\epsilon = 0.95$; $\gamma = 72°$

(c) Skewness – real Morlet $\boldsymbol{k} = (10, 0)^T$; $\gamma = 72°$

(d) Kurtosis – Mexican hat $\epsilon = 0.00$

(e) Kurtosis – Mexican hat $\epsilon = 0.95$; $\gamma = 108°$

(f) Kurtosis – real Morlet $\boldsymbol{k} = (10, 0)^T$; $\gamma = 72°$

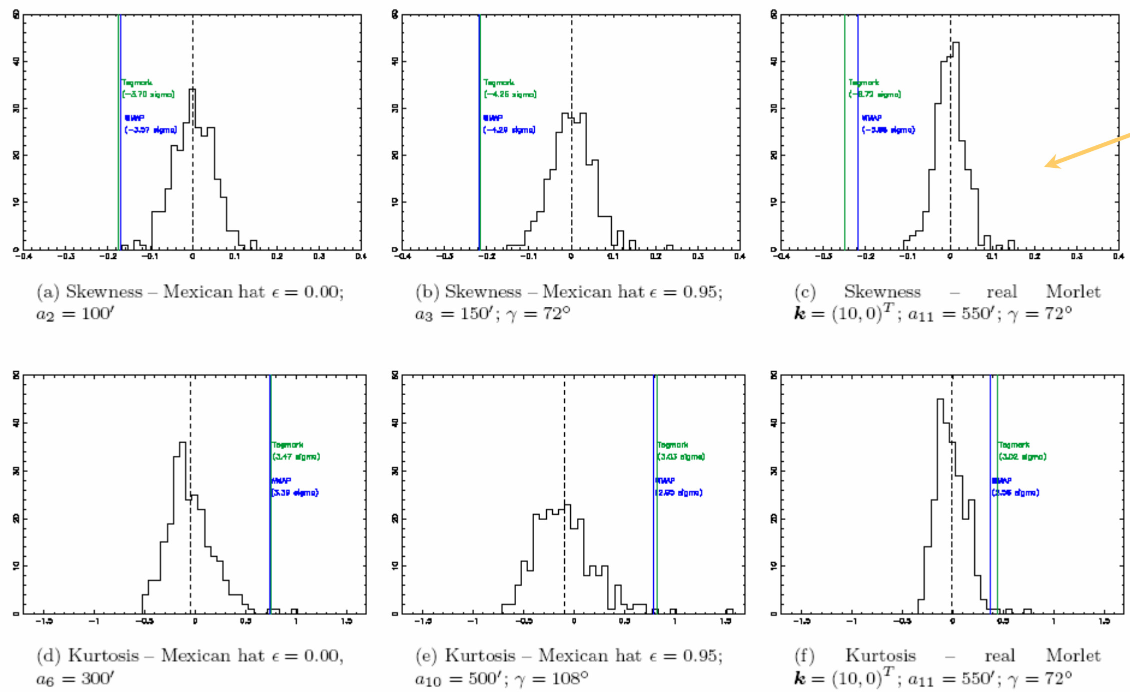
Figure 6. Spherical wavelet coefficient statistics for each wavelet. Confidence regions obtained from 250 Monte Carlo simulations are shown for 68% (red), 95% (orange) and 99% (yellow) levels, as is the mean (solid white line). Only the orientations corresponding to the most significant deviations from Gaussianity are shown for the Mexican hat $\epsilon = 0.95$ and real Morlet wavelet cases.

Statistical significance

- Analyse most significant detections in more detail

WMAP: 5.9σ

Tegmark: 6.7σ



(a) Skewness – Mexican hat $\epsilon = 0.00$; $a_2 = 100'$

(b) Skewness – Mexican hat $\epsilon = 0.95$; $a_3 = 150'$; $\gamma = 72°$

(c) Skewness – real Morlet $\boldsymbol{k} = (10, 0)^T$; $a_{11} = 550'$; $\gamma = 72°$

(d) Kurtosis – Mexican hat $\epsilon = 0.00$, $a_6 = 300'$

(e) Kurtosis – Mexican hat $\epsilon = 0.95$; $a_{10} = 500'$; $\gamma = 108°$

(f) Kurtosis – real Morlet $\boldsymbol{k} = (10, 0)^T$; $a_{11} = 550'$; $\gamma = 72°$

Figure 7. Histograms of spherical wavelet coefficient statistic obtained from 250 Monte Carlo simulations. The mean is shown by the dashed vertical line. The observed statistics for the WMAP and Tegmark maps are shown by the blue and green lines respectively. The number of standard deviations these observations deviate from the mean is also displayed on each plot. Only those scales and orientations corresponding to the most significant deviations from Gaussianity are shown for each wavelet.

Statistical significance

- Consider statistical significance of results as a whole (adopting the most conservative approach)
- True significance of some detections outside 99% CL is considerably lower when all statistics are taken into account
- 5.9σ detection found with real Morlet wavelets (at size on sky of $\sim 26°/3°$, orientation 72°) occurs at *99% significance level*
- Alternatively, consider all statistics simultaneously using χ^2 test

Table 2. Deviation and significance levels of spherical wavelet coefficient statistics calculated from the WMAP map

	Skewness ($a_2 = 100'$)	Kurtosis ($a_6 = 300'$)
N_σ	-3.57	3.39
$N_{\rm dev}$	5 maps	7 maps
δ	98%	97%

(a) Mexican hat $\epsilon = 0.00$

	Skewness ($a_3 = 150'$, $\gamma = 72°$)	Kurtosis ($a_{10} = 500'$; $\gamma = 108°$)
N_σ	-4.29	2.95
$N_{\rm dev}$	8 maps	55 maps
δ	97%	78%

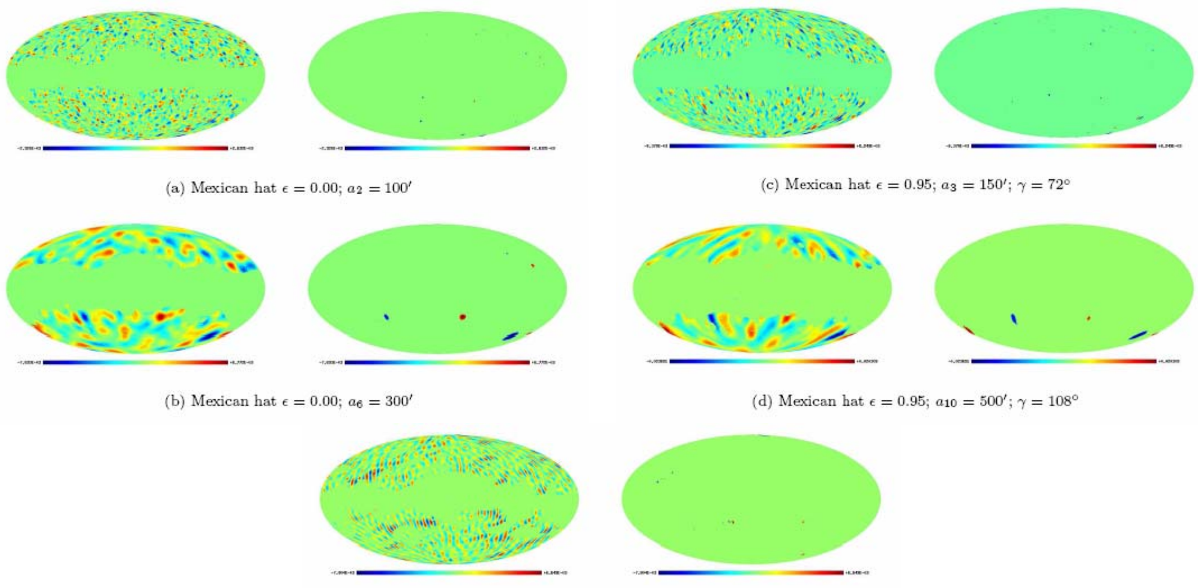
(b) Mexican hat $\epsilon = 0.95$

	Skewness ($a_{11} = 550'$; $\gamma = 72°$)	Kurtosis ($a_{11} = 550'$; $\gamma = 72°$)
N_σ	-5.88	2.56
$N_{\rm dev}$	3 maps	167 maps
δ	99%	33%

(c) Real Morlet $\boldsymbol{k} = (10, 0)^T$

Localised deviations

- Wavelets inherently provide *spatial localisation*
- Non-Gaussian *sources detected*



(a) Mexican hat $\epsilon = 0.00$; $a_2 = 100'$

(c) Mexican hat $\epsilon = 0.95$; $a_3 = 150'$; $\gamma = 72°$

(b) Mexican hat $\epsilon = 0.00$; $a_6 = 300'$

(d) Mexican hat $\epsilon = 0.95$; $a_{10} = 500'$; $\gamma = 108°$

(e) Real Morlet $\boldsymbol{k} = (10, 0)^T$; $a_{11} = 550'$; $\gamma = 72°$

Figure 8. Spherical wavelet coefficient maps (left) and thresholded maps (right). To localise most likely deviations from Gaussianity on the sky, the coefficient maps exhibiting strong non-Gaussianity are thresholded so that only those coefficients above 3σ (in absolute value) are shown. Due to the apparent similarity of the WMAP team and Tegmark maps, only coefficients for the analysis of the WMAP map are shown above.

Conclusions

- *Fast directional CSWT* analysis of *WMAP 1-year data*
- Confirmed the results obtained by Vielva et al. (2003) & made a number of additional detections of non-Gaussianity
- *Detection of 5.9σ* (6.7σ for Tegmark et al. map) at *99% significance* found using real Morlet wavelets at size on sky $\sim$26°/3°
- Non-Gaussian *sources localised*
- Future work
  - Further *statistical analysis*
  - *Examine localised regions* to determine origin of non-Gaussianity
  - Perform analysis on *COBE-DMR data* to see if detection of cosmic origin
  - Apply directional CSWT to *detection of orientated objects*, e.g. elliptical clusters, cosmic strings(?!)