CosmoInformatics

Novel spherical informatics techniques for studying cosmic evolution

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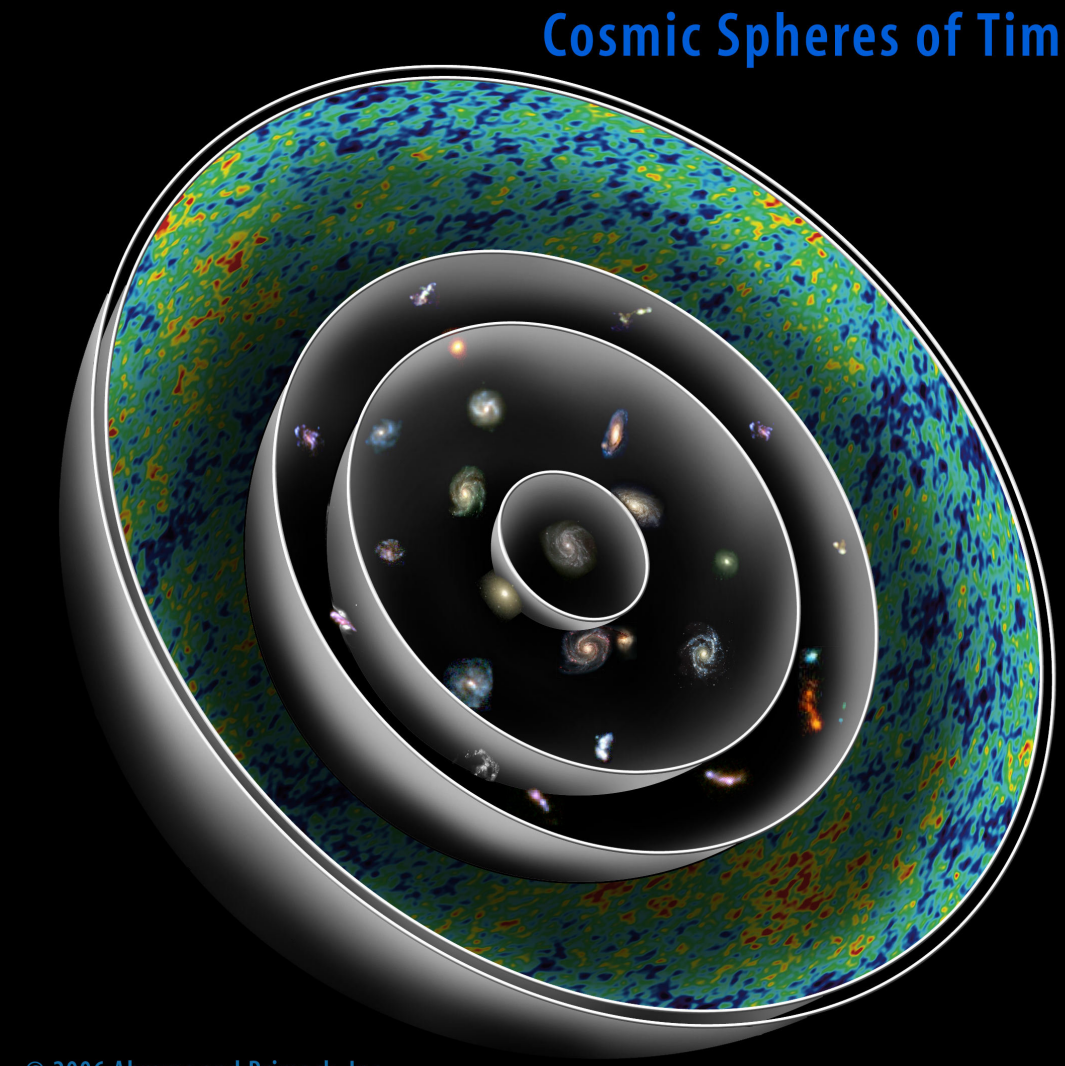
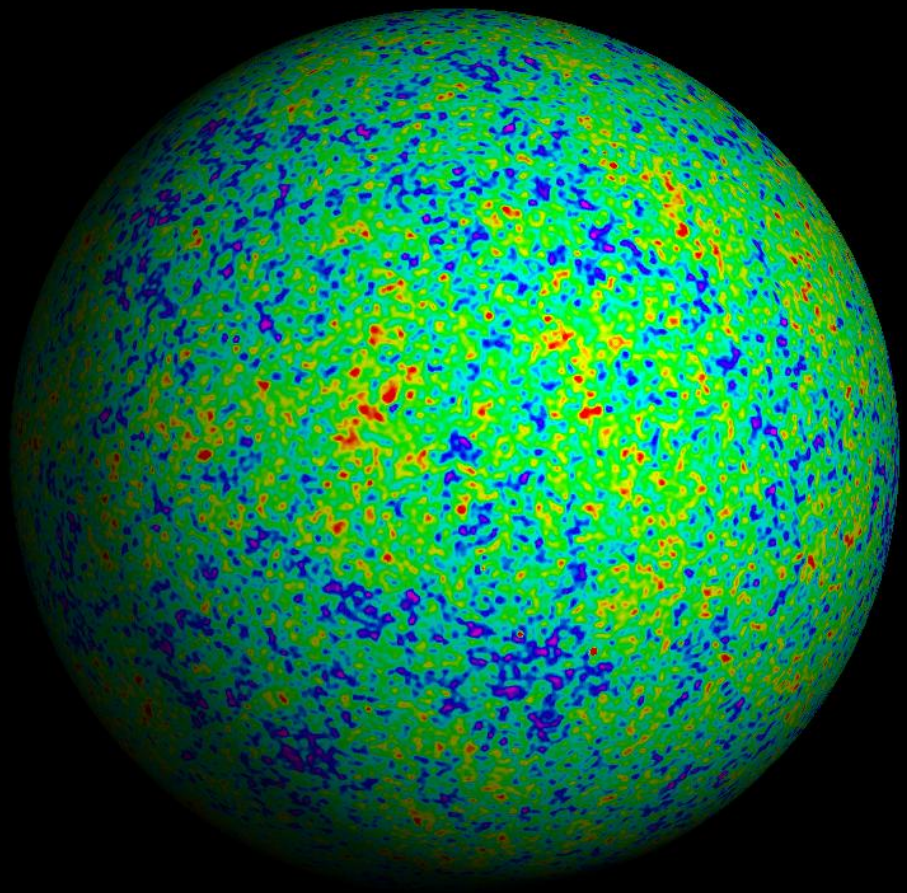
UCL Data Intensive Science CDT Research Festival, 9 June 2017

Observations made on the celestial sphere



Cosmic microwave background (CMB) on the celestial sphere

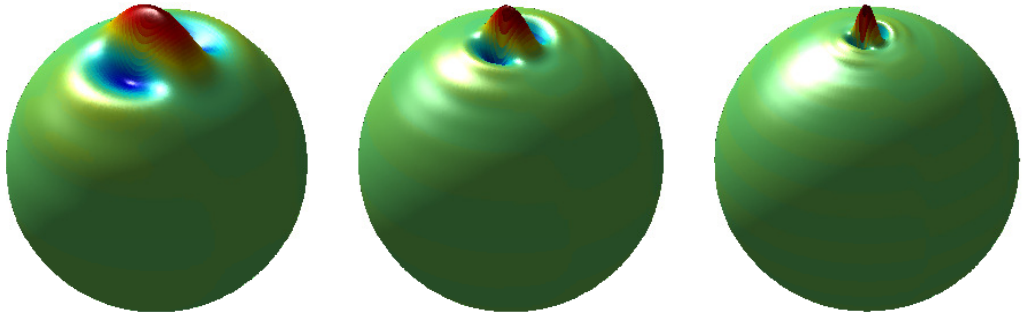


Credit: WMAP

Wavelets on the sphere

- Spin scale-discretised wavelet transform given by projection onto each wavelet (McEwen *et al.* 2015; McEwen 2015; McEwen *et al.* 2013; Wiaux, McEwen *et al.* 2008):

$$\underbrace{W^{s\Psi^j}(\rho) = \langle {}_sf,\, \mathcal{R}_\rho\, {}_s\Psi^j \rangle}_{\text{projection}} = \int_{\mathbb{S}^2} \mathrm{d}\Omega(\theta,\varphi)\, {}_sf(\theta,\varphi)\, (\mathcal{R}_\rho\, {}_s\Psi^j)^*(\theta,\varphi)\,.$$



(a) $j = 3$ (b) $j = 4$ (c) $j = 5$

Figure: Wavelets on sphere

- Original function may be recovered exactly in practice from wavelet coefficients:

$${}_sf(\omega) = \underbrace{\sum_{j=0}^{J}}_{\text{finite sum}} \underbrace{\int_{\mathrm{SO}(3)} \mathrm{d}\varrho(\rho)\, W^{s\Psi^j}(\rho)\, (\mathcal{R}_\rho\, {}_s\Psi^j)(\omega)}_{\text{wavelet contribution}}\,.$$

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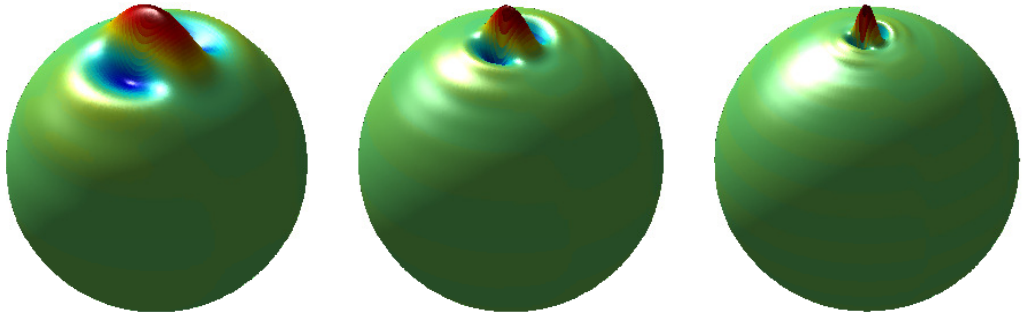
projection



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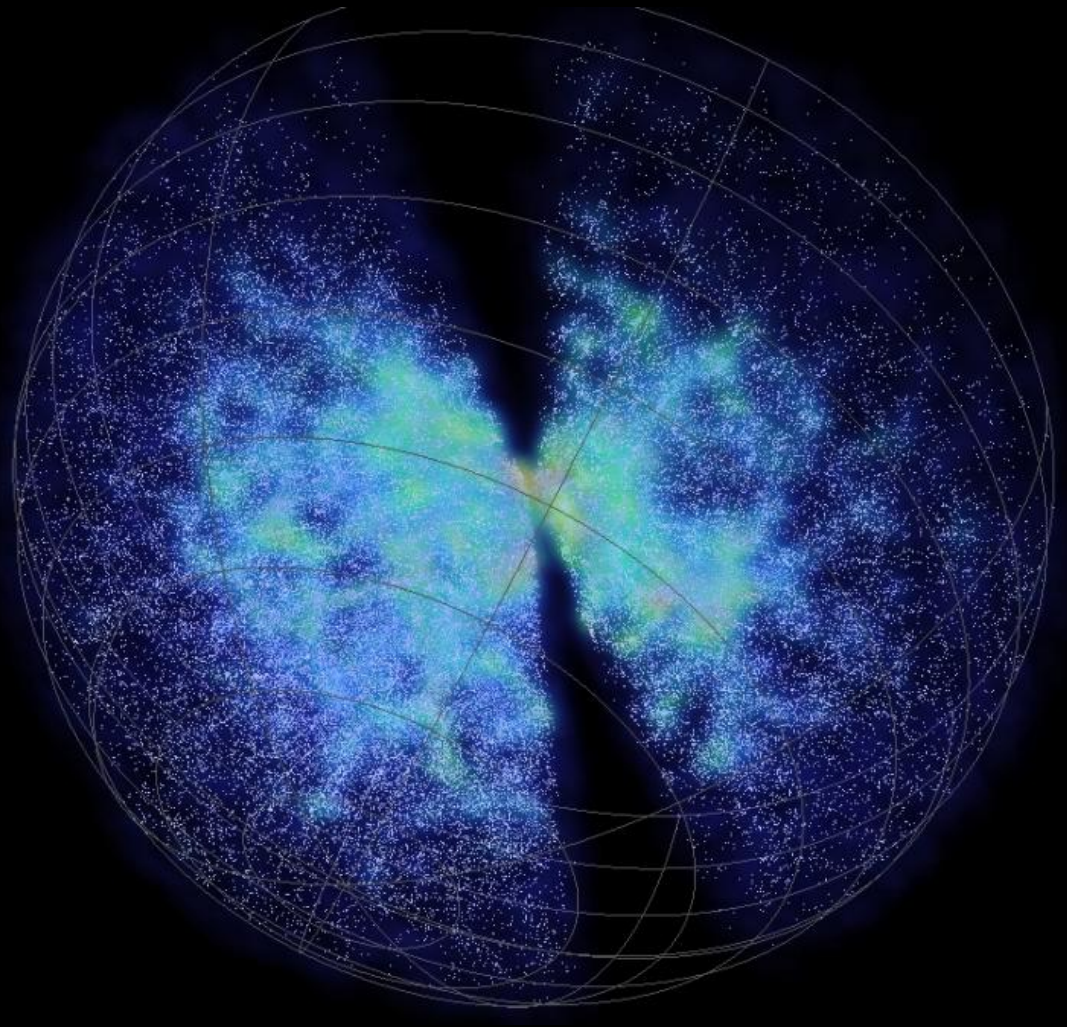
finite sum

wavelet contribution

Galaxy distribution tracing large-scale structure on the 3D ball



Credit: SDSS

Fourier-LAGuerre wavelets (flaglets) on the ball

- Fourier-Laguerre wavelet (flaglet) transform is given by the projection onto each wavelet (Leistedt & McEwen 2012):

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- Opens up wavelet analyses of galaxy distribution tracing the large-scale structure (LSS).

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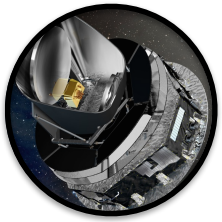
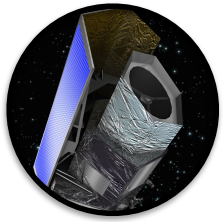
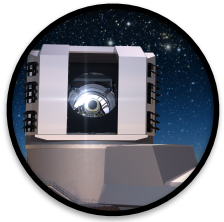
Understanding cosmic evolution and structure



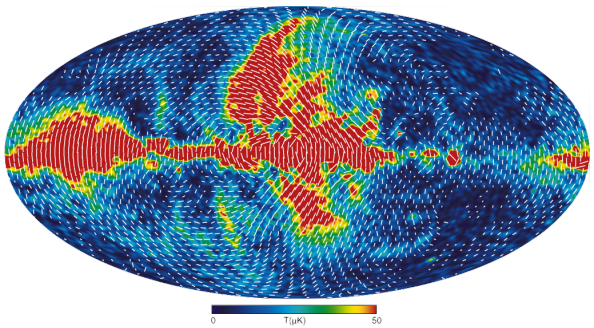
Planck



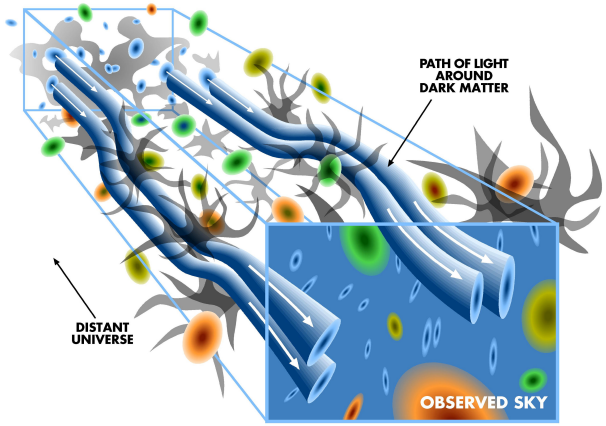
Euclid



LSST



(a) CMB polarization
[Credit: WMAP]



(b) Weak gravitational lensing
[Credit: Tyson]

Figure: Cosmological probes

- What is the energy scale of inflation?
- What happened following inflation?
- What is the nature of dark energy?
- Where is the dark matter?