Bianchi VII_h signatures and WMAP

Evidence for universal shear and rotation?

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Outline

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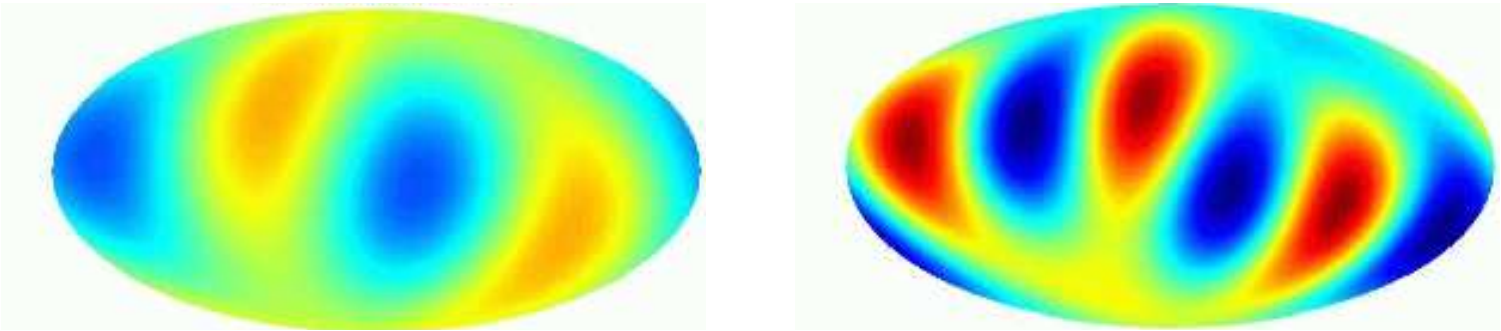
Multipole alignments

- Peculiar planarity and alignment of quadrapole and octopole (de Oliveira-Costa et al. 2003; Copi et al. 2004; etc.) (and also between some other low ℓ's)



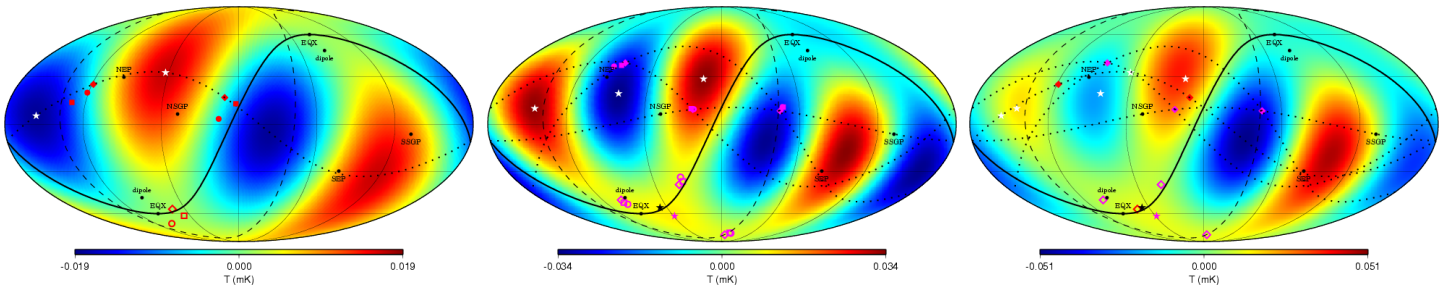
(a) Quadrapole ($\ell = 2$) (b) Octopole ($\ell = 3$)

Figure: Multipole maps

- Some works claim that planar shape is not statistically significant (e.g. Slosar & Seljak 2005; Land & Magueijo 2005) but consensus is that alignment is peculiar (using range of tests)
- Infamously dubbed the Axis of Evil (AoE)

Multipole alignments

- Various works claim close alignment with ecliptic and/or dipole (e.g. Copi et al. 2006)



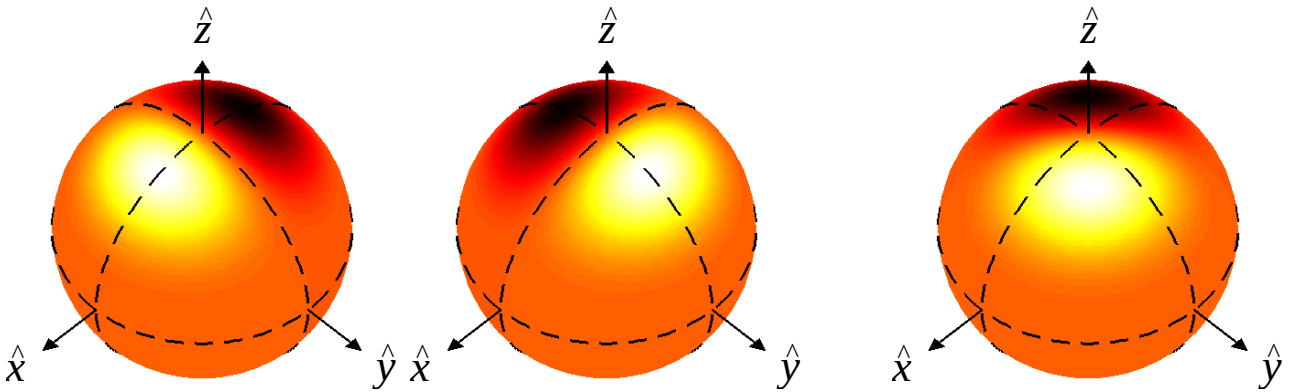
(a) Quadrapole (b) Octopole (c) Quadrapole + Octopole

Figure: Quadrapole and octopole alignments with ecliptic and dipole

Steerable wavelets on the sphere

- Steerable wavelets may be expressed as a linear combination of a finite number of basis wavelets

$$[\mathcal{R}_{(\alpha=0,\beta=0,\gamma)}\Psi](\omega) = \sum_{m=1}^{M} k_m(\gamma)\, \Psi_m(\omega)$$



(a) Basis wavelets (b) Interpolated wavelet

Figure: First derivative of Gaussian on the sphere (Wiaux et al. 2005a)

- Thus wavelet coefficients for any orientation may be derived from coefficients computed for a small number of basis orientations

Steerable wavelet anisotropy test: Method

- Test methodology (Wiaux et al. 2005b, Vielva et al. 2006)
  - Use steerable wavelets to pick out preferred orientation
  - Increment weighted votes for all points on great circle
  - Construct map giving probability a given pixel is *seen* by local CMB features

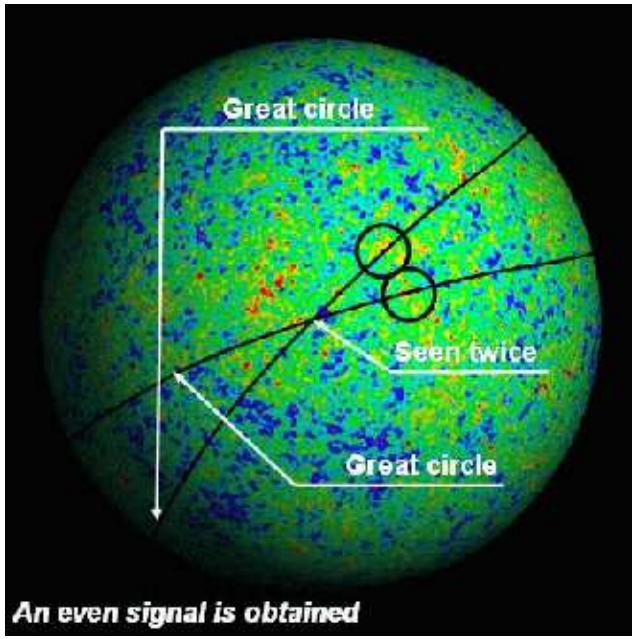
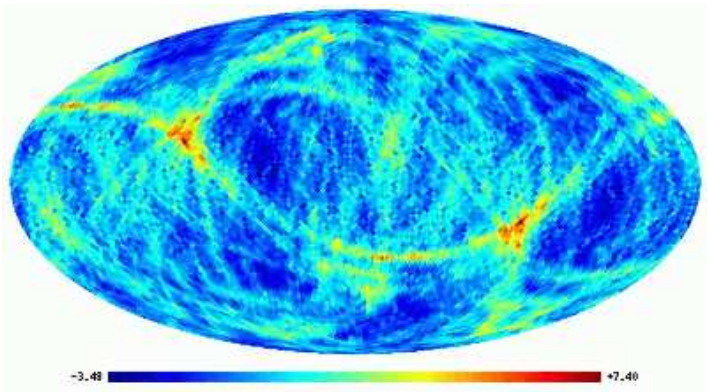


Figure: Illustration of steerable wavelet anisotropy test

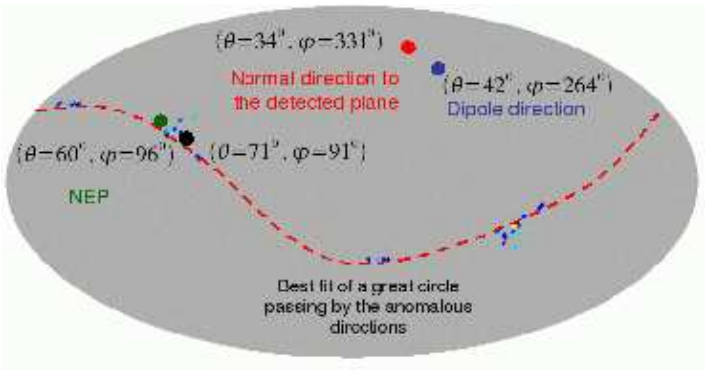
- Analysis run on WMAP data using second derivative of Gaussian

Steerable wavelets anisotropy test: Results

- Anisotropy map shows deviations from anisotropy relative to Monte Carlo simulations
- Pick out great circle, with pole very close to dipole



(a) Wavelet anisotropy map



(b) Anomalous directions

Figure: Steerable wavelet anisotropy results

North-south power asymmetry

- ML estimate of local angular power spectrum on small patches (Eriksen et al. 2004)
- Amplitude of disks in the northern Galactic hemisphere generally lower than in simulated maps; amplitude of disks in the southern Galactic hemisphere generally higher than in simulated maps

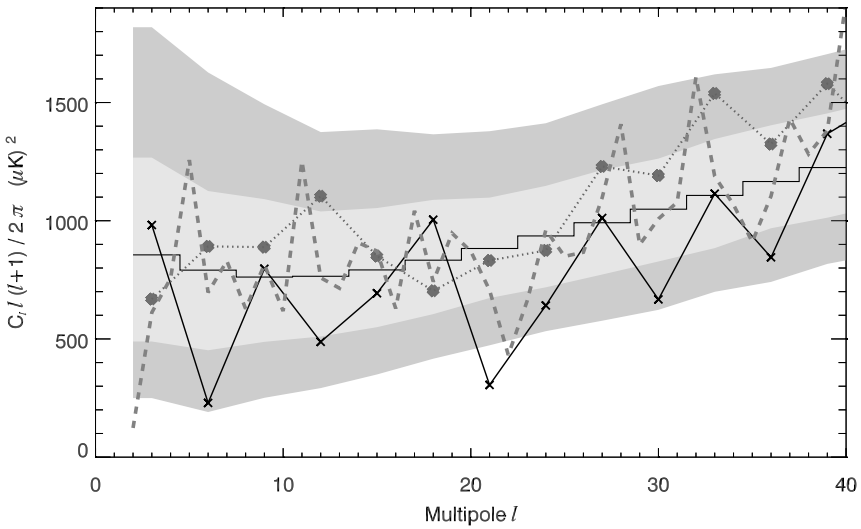


Figure: Power spectrum comparison

North-south power asymmetry

- Colour of disks indicates power ratio relative to overall power
- Axis of maximum asymmetry found to be close to ecliptic

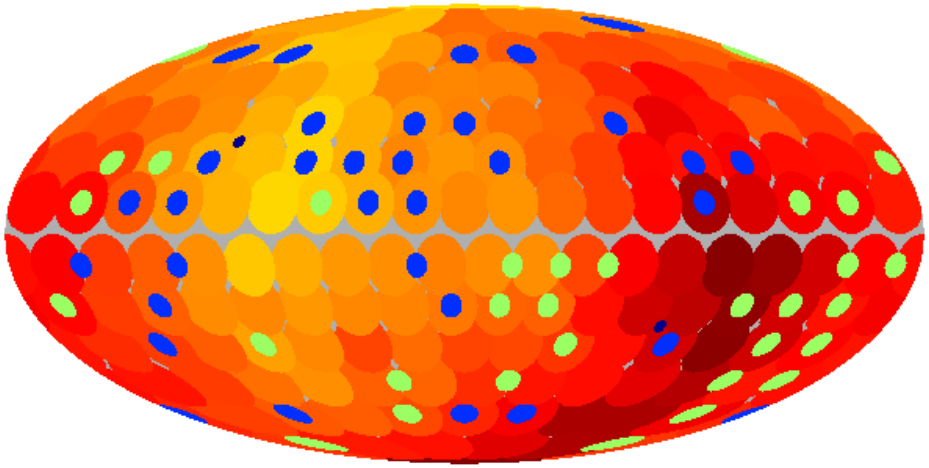


Figure: Local power spectrum analysis

Cold spot

- Deviations from Gaussianity detected in kurtosis of spherical Mexican hat wavelet (SMHW) (Vielva et al. 2004)

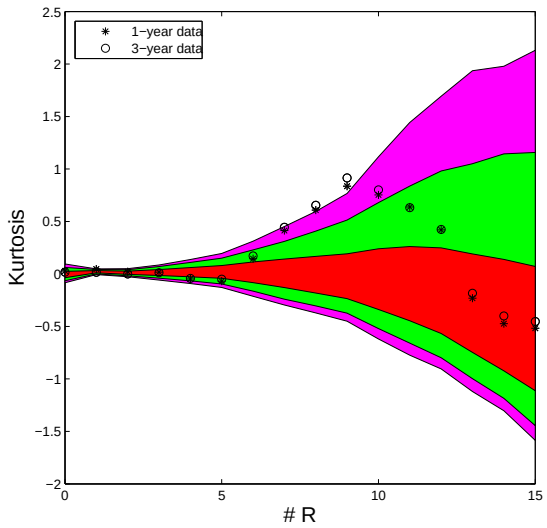
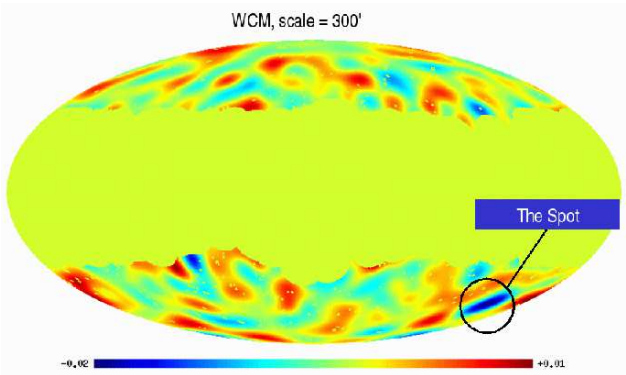


Figure: SMHW kurtosis

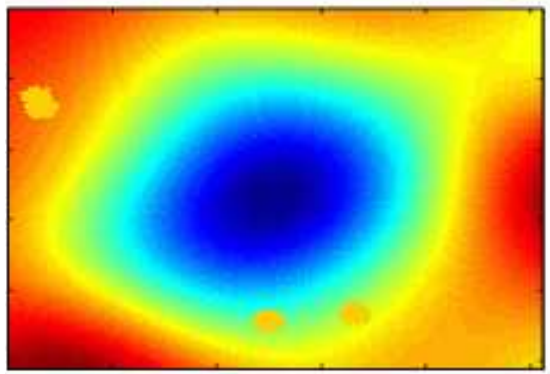
- Large non-Gaussian cold spot detected
- Various test statistics indicate extremely large and cold spot unlikely at >99% level (Cruz et al. 2004, 2006a, 2006b)

Cold spot

- Morphology – approximately circular
- Excluding the spot the data are consistent with Gaussianity (using the SMHW kurtosis test)
- Not systematics, not foregrounds
- Origin?



(a) SMHW coefficients



(b) $22^\circ \times 22^\circ$ patch

Figure: Cold spot

Outline

Bianchi cosmologies

- Unexplained anomalies rekindled interest in Bianchi type cosmologies
- Bianchi class of universes are anisotropic, spatially homogeneous generalisations of the Friedman universes, some of which contain vorticity and shear
- In flat, open, and closed universes the Bianchi models are VII_0, VII_h and IX respectively
- Bianchi models induce characteristic signatures in CMB (only temperature fluctuations considered to date)
- Bianchi VII_h cosmology exhibits richest structure and has been examined exclusively in recent works
- (Note that only the late-time effects of the rotation on photons since last scattering are examined)

Bianchi parameters

- "Tightness" of spiral parameterised by h or x, where x defines the scale on which the principle axes of shear and rotation change orientation

$$x = \sqrt{\frac{h}{1 - \Omega_{\text{total}}}}$$

- Amplitude may be parameterised by vorticity or shear

$$\left(\frac{\omega}{H}\right)_0 = \frac{(1+h)^{1/2}(1+9h)^{1/2}}{3\sqrt{2}x^2\Omega_{\text{total}}}\left(\frac{\sigma}{H}\right)_0$$

- Handedness κ (right hand spiral $\Rightarrow \kappa = +1$; left hand spiral $\Rightarrow \kappa = -1$)
- Position and orientation arbitrary; parameterise by Euler angles $\rho = (\alpha, \beta, \gamma) \in \mathrm{SO}(3)$

Bianchi signatures

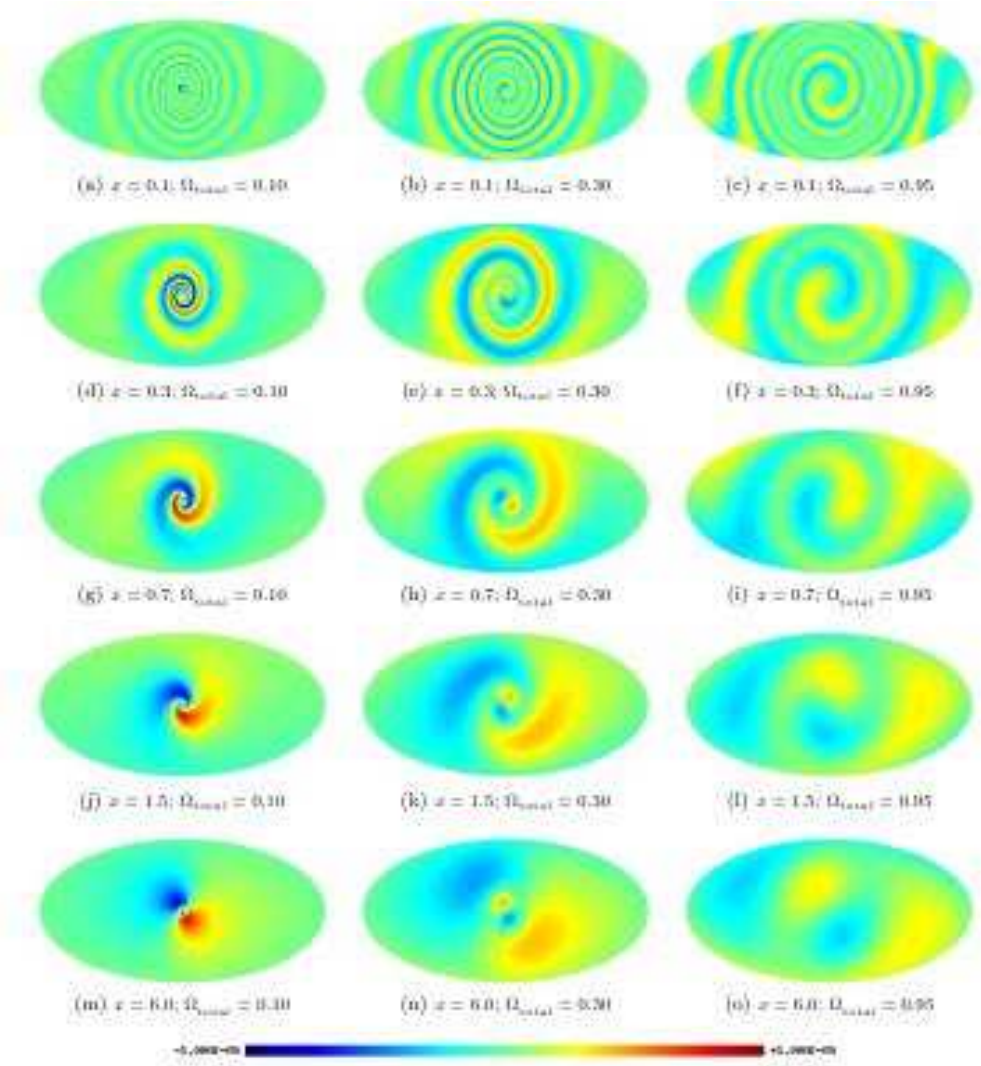
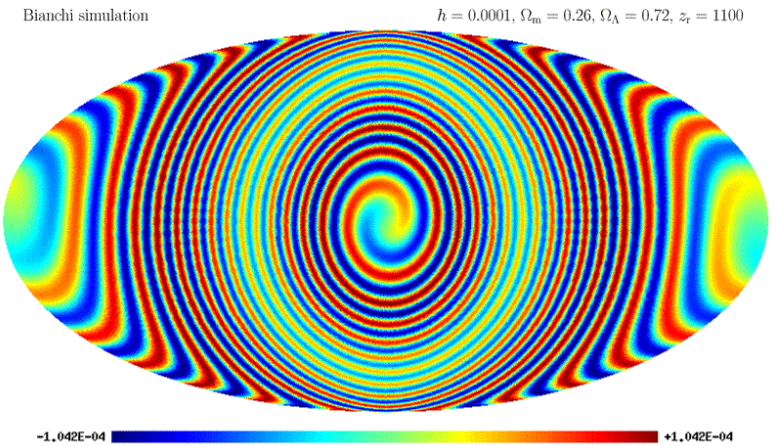
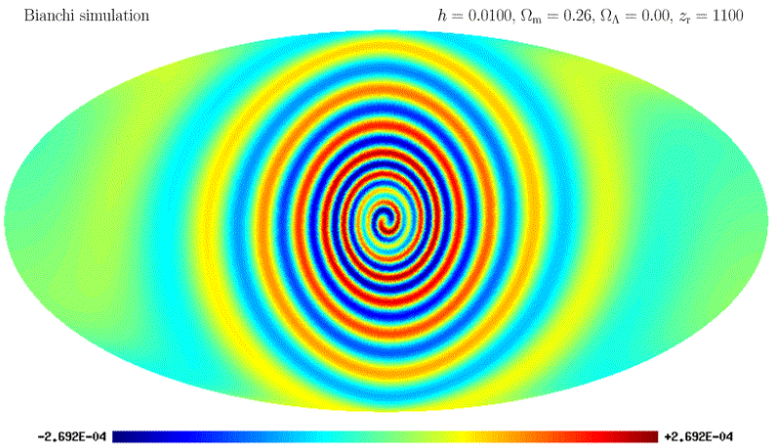


Figure 5.1: Simulated $\Delta T/T_0$ Bianchi VII_0 maps for a range of Bianchi parameters. In these maps the swirl pattern typical of Bianchi-induced temperature fluctuations is rotated from the south pole to the Galactic centre for illustrational purposes. The other parameters used for generating these maps are as follows: $\left(\frac{\omega}{H}\right)_0 = 10^{-10}$; $z_{\mathrm{ls}} = 1000$; $\kappa = +1$; $(\alpha, \beta, \gamma) = (0^\circ, -90^\circ, 0^\circ)$.

Bianchi signatures



(a) Varying h



(b) Varying Ω_Λ

Figure: Animations of Bianchi signatures for varying parameters (click for animations)

History

- Barrow *et al.* (1985):
  - Derived Bianchi signatures induced in CMB (neglecting dark energy as it was not considered plausible at the time)
- Bunn *et al.* (1996):
  - COBE-DMR data $\rightarrow \left(\frac{\omega}{H}\right)_0 < 3 \times 10^{-7}$
- Kogut *et al.* (1997):
  - COBE-DMR data $\rightarrow \left(\frac{\omega}{H}\right)_0 < 6 \times 10^{-8}$
- Jaffe *et al.* (2005; 2006a; 2006b; 2006c):
  - WMAP data
  - Find a statistically significant template with $\left(\frac{\omega}{H}\right)_0 = 9.5 \times 10^{-10}$
  - BUT inconsistent with concordance cosmology
- Lasenby *et al.* (in preparation):
  - Derived Bianchi signatures induced in CMB in presence of dark energy
- Bridges *et al.* (2006):
  - Bayesian analysis using MCMC sampling
  - Compute Bayesian evidence
  - (repeated recently using nested sampling $\rightarrow$ improved accuracy)

Outline

Best fit template

- Statistically significant template found in WMAP by Jaffe *et al.* (2005, 2006a)
- Parameter estimation performed using semi-analytic plus grid evaluation technique
- Statistical significance determined using Monte Carlo simulations
- (Readdress template fitting and model selection problem later)

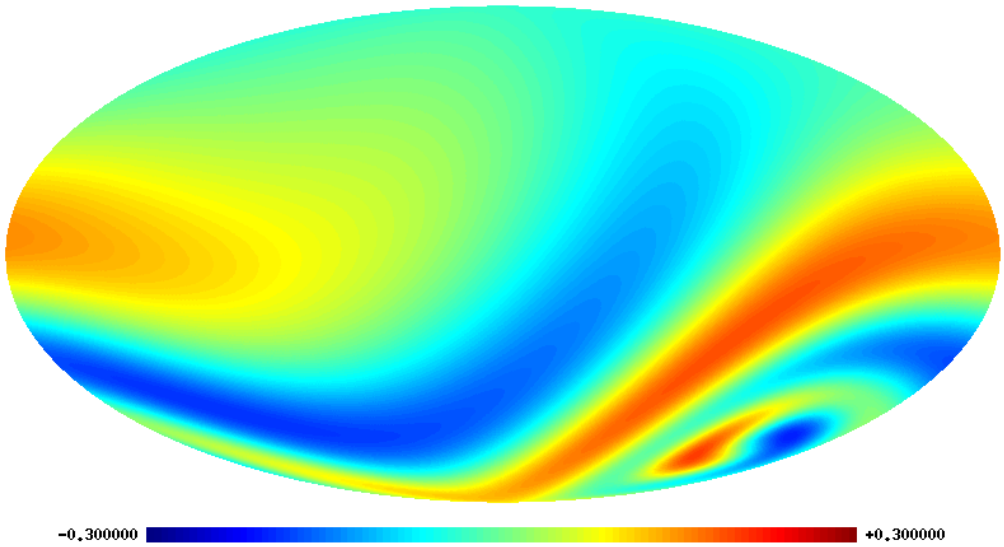


Figure: Best fit Bianchi template found by Jaffe *et al.* (click for animation)

Bianchi correction

- Use best-fit Bianchi template to ‘correct’ WMAP data
- Investigate impact on previously reported anomalies reported in WMAP data. . .

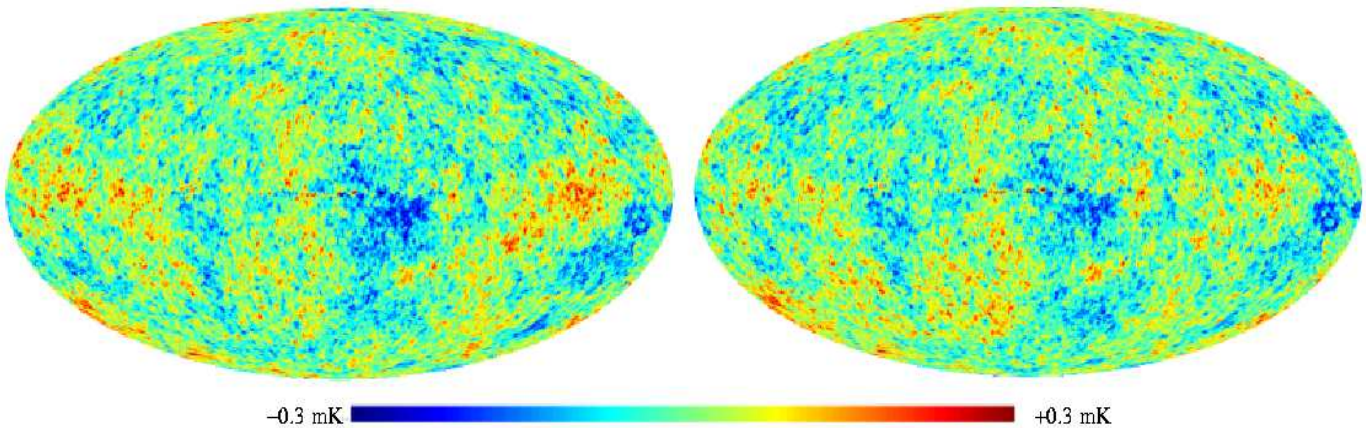
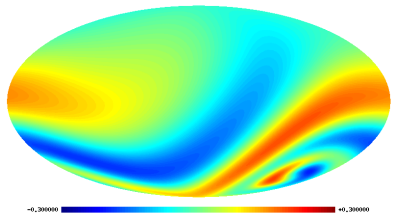


Figure: Original (left) and Bianchi corrected (right) ILC map (click for animation)



Jaffe *et al.* best fit template
(same color bar but scaled by 4)

Implications for WMAP anomalies

- Multipole alignments disappear or are significantly mitigated

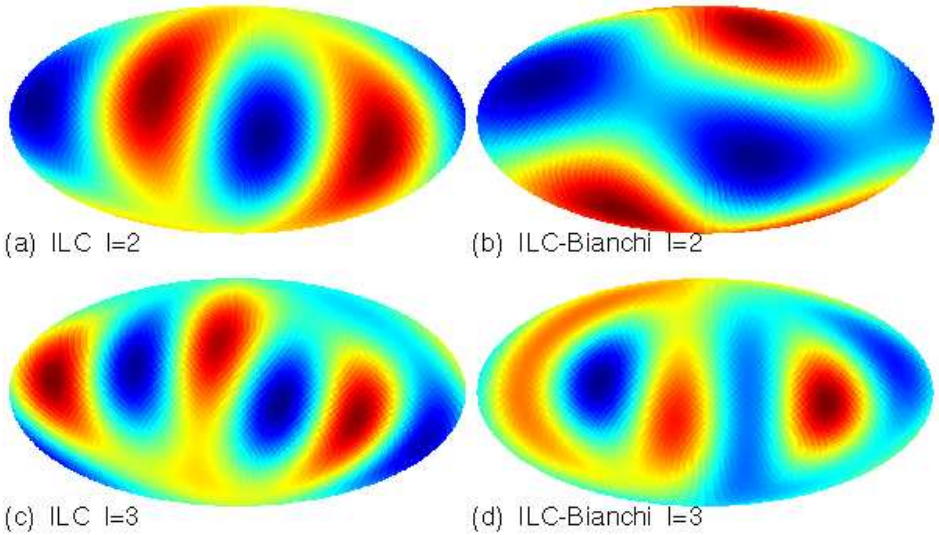


Figure: Quadrapole and octopole

- Steerable wavelet isotropy not yet repeated on Bianchi corrected data

Implications for WMAP anomalies

- Significance of any power asymmetry drops from 99.3% to 86.4%

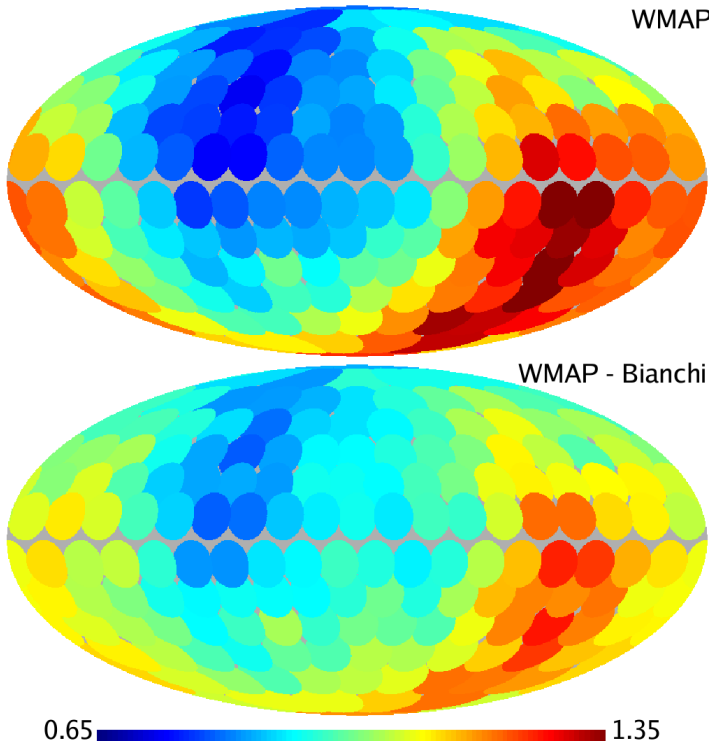


Figure: Power asymmetry

Implications for WMAP anomalies

- SMHW kurtosis is essentially compatible with Gaussianity
- Cold spot may drive best-fit Bianchi template?

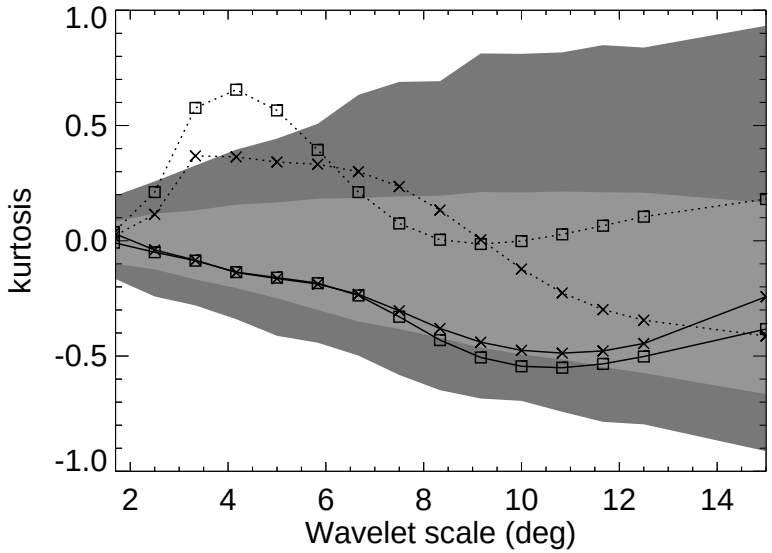


Figure: SMHW kurtosis

- [Detection of non-Gaussian made using real Morlet wavelet (McEwen *et al.* 2005) remains (McEwen *et al.* 2006)]

Outline

Motivation

- Best-fit template found by Jaffe *et al.* is inconsistent with concordance cosmology
- Extend analysis to incorporate dark energy (concurrent dark energy fit performed by Jaffe *et al.* (2006c))
- Rigorous parameter estimation (investigate Ω_m–Ω_Λ degeneracy)
- Rigorous model selection using Bayesian evidence
- Work done primarily by Michael Bridges with Anthony Lasenby, Mike Hobson & myself (Bridges 2006)

Parameter estimation

- Parameter estimation adopted by Jaffe *et al.* performed using semi-analytic plus grid evaluation technique
- Simultaneously fit Bianchi component *and* cosmology
- Bayes theorem

$$P(\Theta|\boldsymbol{d}, M) = \frac{P(\boldsymbol{d}|\Theta, M)\, P(\Theta|M)}{P(\boldsymbol{d}|M)}$$

- Likelihood (assuming Gaussianity)

$$P(\{d_{\ell m}\}|\Theta_{\rm B}, \Theta_{\rm C}) \propto \prod_{\ell} \frac{1}{\sqrt{C_\ell(\Theta_{\rm C})}} \mathrm{e}^{-\frac{[d_{\ell 0} - t_{\ell 0}(\Theta_{\rm B})]^2}{C_\ell(\Theta_{\rm C})}} \prod_{m} \frac{2}{C_\ell(\Theta_{\rm C})} \mathrm{e}^{-\frac{|d_{\ell m} - t_{\ell m}(\Theta_{\rm B})|^2}{C_\ell(\Theta_{\rm C})}}$$

- Use Markov chain Monte Carlo (MCMC) sampling to sample directly from posterior from which one may obtain parameter estimates

Bayesian evidence

- Bayesian evidence

$$E = P(\boldsymbol{d}|M) = \int P(\boldsymbol{d}|\Theta, M)\, P(\Theta|M)\, \mathrm{d}\Theta\,,$$

- Naturally incorporates Occam's razor, trading off model simplicity and accuracy
- Compare evidence ratios to determine preferred model (use Jeffreys scale to rank):
  - $\Delta \ln E < 1$ is inconclusive;
  - $1 < \Delta \ln E < 2.5$ is significant
  - $2.5 < \Delta \ln E < 5$ is strong
  - $\Delta \ln E > 5$ is conclusive
- Challenging to evaluate numerically
  - Thermodynamic integration (see *e.g.* Bridges (2005))
  - Nested-sampling (Shaw, Bridges & Hobson (in preparation))
- Parameter estimation and model selection validated on simulated data

Results: Bianchi degeneracy

- Inclusion of Ω_Λ introduces a marked degeneracy with Ω_m
- Not possible to find a Bianchi template consistent with concordance cosmology (at 1σ level)

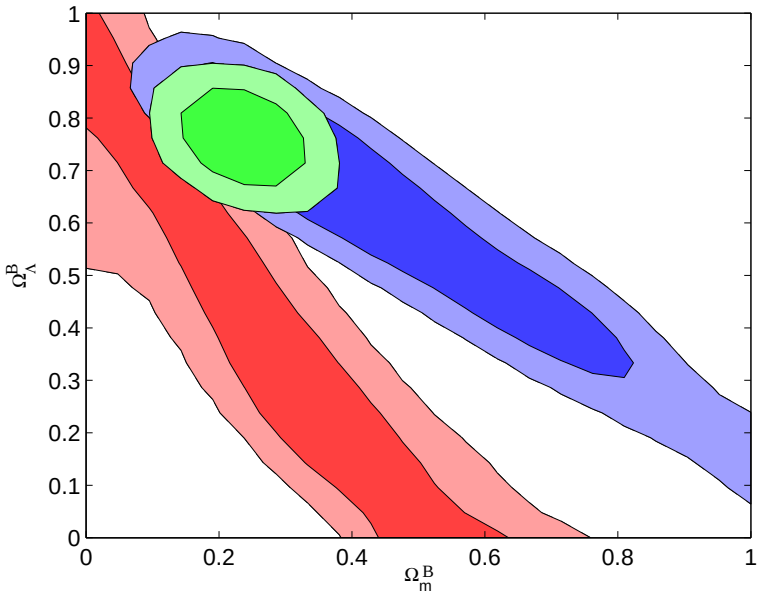


Figure: Bianchi template degeneracy

- Decouple Bianchi parameters and cosmology and consider as phenomenological template only (also set $\Omega_\Lambda = 0$ since including Λ doesn't alter morphology of allowable templates)

Results: Best-fit left handed model

- Find best-fit left handed model very similar to that found by Jaffe *et al.*

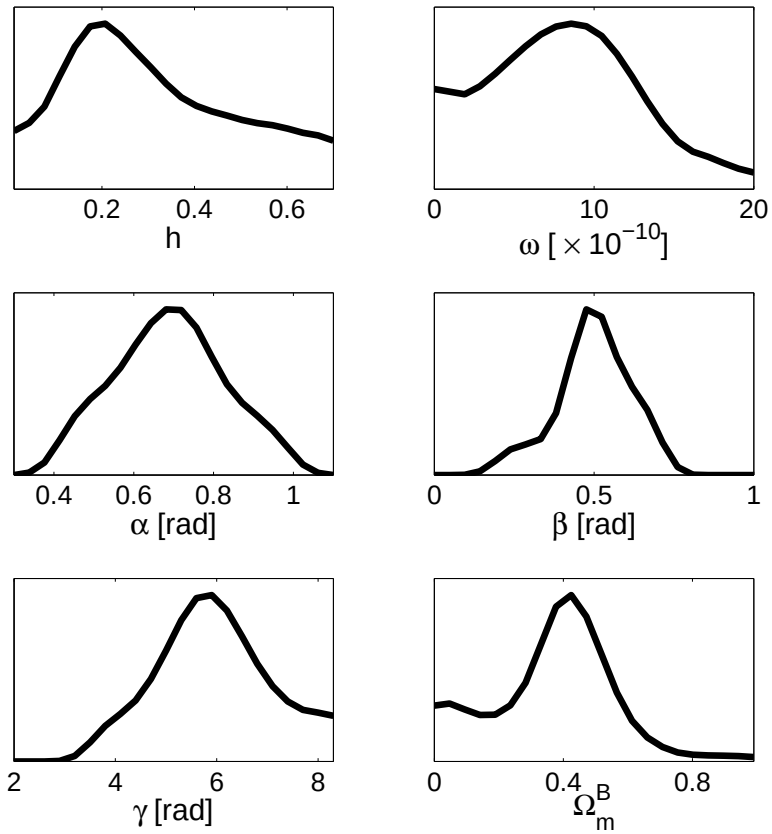


Figure: Marginalised posteriors for Bianchi parameters of left handed model

Results: Best-fit right handed model

- No suitable right handed model

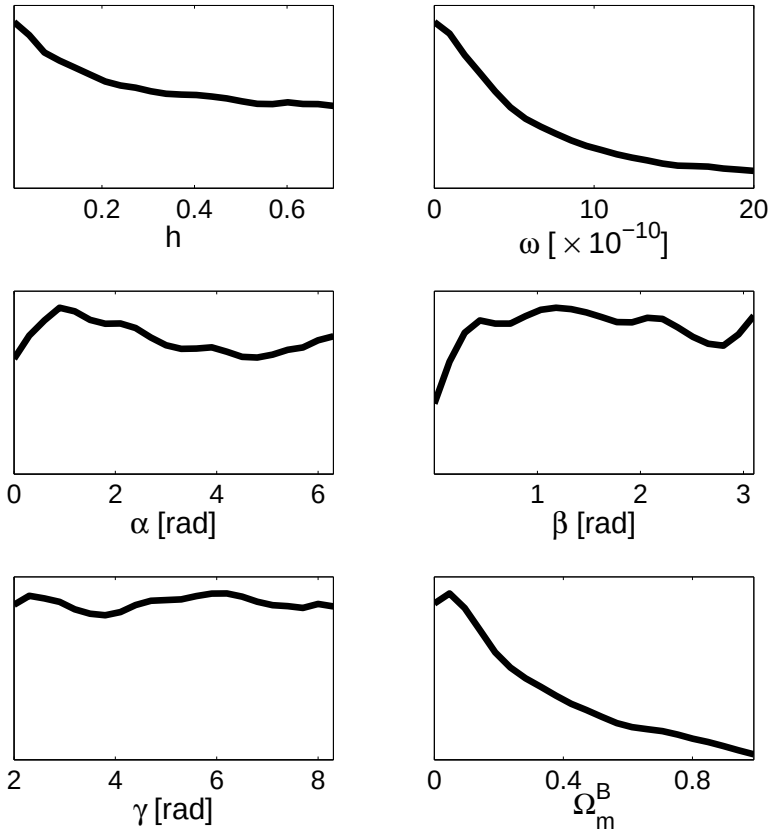


Figure: Marginalised posteriors for Bianchi parameters of righ handed model

Results: Bayesian evidence

- Computed originally in Bridges (2006) using thermodynamic integration but with insufficient burn-in
- Recomputed using nested sampling $\longrightarrow$ improved accuracy (in preparation)
- Right handed models $\Delta \ln E \lesssim -1$ for range of priors and ILC1 and ILC3 data $\Rightarrow$ disfavoured significantly
- Left handed models $-1 \lesssim \Delta \ln E \lesssim 0$ for range of priors and ILC1 and ILC3 data $\Rightarrow$ inconclusive evidence (but unlikely)

Outline

Summary

- Bianchi $\mathrm{VII_h}$ models inconsistent with concordance cosmology
- Bianchi parameters detached from cosmological ones
- No longer physical motivated
- Left handed template found in data but inconclusive Bayesian evidence to justify or rule out inclusion of Bianchi component
- Nevertheless best-fit template does mitigate many anomalies reported in WMAP data
- Future work
  - Cold spot influence on best-fit Bianchi signature
  - Polarisation in Bianchi models

- To download best-fit templates and code to compute Bianchi signatures see:
  `http://www.mrao.cam.ac.uk/~jdm57/`