# Intrinsic advantages of the $w$ component and spherical imaging for wide-field radio interferometry

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## Abstract

Incorporating wide-field considerations in interferometric imaging is of increasing importance for next-generation radio telescopes. Compressed sensing techniques for interferometric imaging have been extended to wide fields recently, recovering images in the spherical coordinate space in which they naturally live. We review these techniques, highlighting: (i) how the effectiveness of the spread spectrum phenomenon, due to the $w$ component inducing an increase of measurement incoherence, is enhanced when going to wide fields; and (ii) how sparsity is reduced by recovering images directly on the sphere. Both of these properties act to improve the quality of reconstructed images.

## 1 Introduction

We are entering a new era of radio astronomy, with a new generation of radio interferometric telescopes under construction and design. Next-generation radio interferometers, such as the Square Kilometre Array[1] (SKA) [1], will inherently observe very large fields of view. Wide fields introduce two important distinctions to standard interferometric imaging: firstly, interferometric images are inherently spherical and planar projections necessarily introduce distortions; and, secondly, non-zero baseline components in the pointing direction of the telescope, referred to as $w$ components, must be taken into account, introducing a non-negligible modulation of the underlying image. If these contributions are ignored, the images recovered from forthcoming telescopes will not reach their full potential.

The theory of compressed sensing has been applied recently to recover images from simulated measurements taken by radio interferometric telescopes [2, 3]. In these works the effectiveness and flexibility of the compressed sensing approach are demonstrated, resulting in reconstructed interferometric images of superior quality to standard methods. The first work is focused on compressed sensing for reconstruction [2], while the second is focused at the level of acquisition on the spread spectrum phenomenon, due to the $w$ component [3] (these techniques are reviewed in a separate article of the current proceedings [4]). Furthermore, compressed sensing techniques have been developed to successfully extract astronomical signals of interest from interferometric observations corrupted by background contributions [5]. More recently, compressed sensing interferometric imaging techniques have been generalised to a wide field-of-view [6]. In this setting images are recovered directly on the sphere, rather than a tangent plane. Contrary to standard interferometric imaging, this approach provides intrinsic advantages that may be exploited in a compressed sensing framework.

[1] `http://www.skatelescope.org/`

## 2 Spherical radio interferometric imaging

Standard interferometric imaging involves recovering an image from noisy and incomplete Fourier measurements. The resulting ill-posed inverse problem is described by the linear system

$$\boldsymbol{y} = \Phi \boldsymbol{x} + \boldsymbol{n} \,, \tag{1}$$

where the linear measurement operator $\Phi$ relates the underlying image $\boldsymbol{x}$ to the incomplete Fourier measurements taken by the interferometer $\boldsymbol{y}$, in the presence of noise $\boldsymbol{n}$. The measurement operator incorporates the primary beam of the telescope, the $w$ component modulation responsible for the spread spectrum phenomenon [3], the Fourier transform and a masking which encodes the incomplete measurements taken by the interferometer. In the context of compressed sensing, this problem has been solved by applying a prior on the sparsity of the signal in a sparsifying basis $\Psi$ or in the magnitude of its gradient. The underlying image is recovered by solving the Basis Pursuit denoising problem

$$\boldsymbol{\alpha}^\star = \underset{\boldsymbol{\alpha}}{\arg\min} \|\alpha\|_1 \;\text{ such that }\; \|\boldsymbol{y} - \Phi\Psi\boldsymbol{\alpha}\|_2 \leq \epsilon \,, \tag{2}$$

where the image is synthesising by $\boldsymbol{x}^\star = \Psi\boldsymbol{\alpha}^\star$, or by solving the Total Variation (TV) problem

$$\boldsymbol{x}^\star = \underset{\boldsymbol{x}}{\arg\min} \|\boldsymbol{x}\|_{\mathrm{TV}} \;\text{ such that }\; \|\boldsymbol{y} - \Phi\boldsymbol{x}\|_2 \leq \epsilon \,, \tag{3}$$

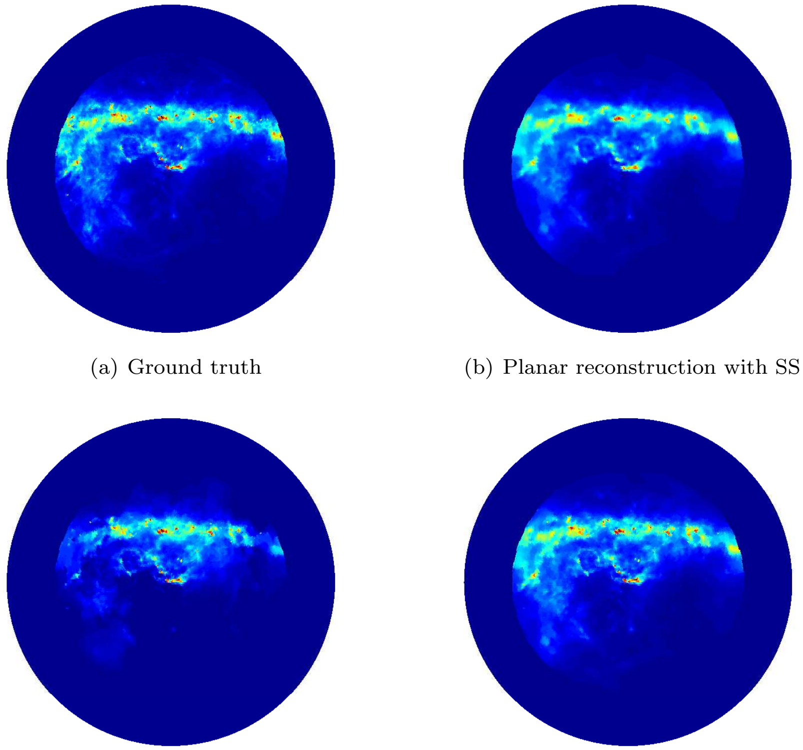
respectively. Recall that the $\ell_1$-norm $\|\cdot\|_1$ is simply given by the sum of the absolute values of the elements of a vector and the squared $\ell_2$-norm $\|\cdot\|_2^2$ is given by the sum of the squares of the elements of a vector. The TV norm $\|\cdot\|_{\mathrm{TV}}$ is given by the $\ell_1$-norm of the gradient of the signal. The tolerance $\epsilon$ is related to an estimate of the noise variance.

To extend the standard compressed sensing imaging framework to wide fields [6], interferometric images are considered directly on the sphere, rather than the equatorial plane. The measurement operator $\Phi_{\mathrm{s}}$ transforming the image defined on the celestial sphere $\boldsymbol{x}_{\mathrm{s}}$ to measurements $\boldsymbol{y}$, consists of augmenting the usual interferometric measurement operator with an initial projection $\mathbf{P}$ from the sphere to the plane, *i.e.*

$$\boldsymbol{y} = \Phi_{\mathrm{s}} \boldsymbol{x}_{\mathrm{s}} + \boldsymbol{n} \,, \tag{4}$$

where $\Phi_{\mathrm{s}} = \Phi\mathbf{P}$. The initial projection simply corresponds to a change from spherical to Cartesian coordinates, resulting in a framework which remains general and does not rely on any small-field assumptions. However, the projection which implements the change of variable is complicated by the discrete setting and the desire to recover a regular grid on the plane to allow the use of fast Fourier transforms (FFTs). In order to project onto a regular grid on the plane, it is necessary to re-grid the pixelisation on the sphere to recover sample values at spherical positions that project directly onto the planar grid. A convolution on the sphere is incorporated in the projection operator to achieve this re-gridding. The convolutional re-gridding on the sphere is similar to the re-gridding often performed when mapping the measurements observed by an interferometer at continuous coordinates to a regular grid, also to afford the use of FFTs [7]. Careful consideration is also given to samplings on the sphere and plane to ensure that the planar grid is sampled sufficiently to accurately represent the projection of a band-limited signal defined on the sphere. Spherical interferometric images may then be recovered by solving the optimisation problems given by (2) and (3), by replacing the measurement operator $\Phi$ with its spherical equivalent $\Phi_{\mathrm{s}}$.

The performance of compressed sensing reconstruction is driven by two factors: sparsity and coherence. Both of these factors can be enhanced in the wide-field spherical interferometric imaging framework [6]. A signal $\boldsymbol{x}$ is said to be sparse if there exists a sparsifying basis yielding coefficients $\boldsymbol{\alpha} = \Psi^{\mathrm{T}}\boldsymbol{x}$, for which the number of non-zero coefficients is much smaller than the dimensionality of the original signal. The theory of compressed sensing states that the more sparse a signal the fewer measurements required to recover it, or similarly, the better the reconstruction quality for a given number of measurements. By recovering interferometric images on the sphere, distorting projections are eliminated and the sparsity of the signal is enhanced, improving the performance of compressed sensing reconstruction. Coherence between



(a) Ground truth

(b) Planar reconstruction with SS

(c) Spherical reconstruction without SS

(d) Spherical reconstruction with SS

Figure 1: Simulated reconstructions of diffuse interstellar Galactic dust in the presence and absence of the spread spectrum (SS) phenomenon.

the measurement and sparsity bases is also a critical factor driving reconstruction performance: as the coherence between the two bases increases, the reconstruction performance degrades. Incoherence ensures that the measurement basis $\Phi$ cannot sparsely represent the sparsity basis $\Psi$, ensuring that signal content is sufficiently probed by incomplete measurements. The so-called spread spectrum phenomenon [3] arises by relaxing the small-field assumption of standard interferometric imaging, resulting in a non-negligible $w$ component modulation in the measurement operator. This modulation may be seen as a convolution of the Fourier representation of the image, spreading its spectrum and increasing incoherence with the (essentially) Fourier measurement basis. The greater the frequency content of the modulation, the larger the spreading. Since the maximum frequency content of the modulation increases with the field-of-view, the spread spectrum phenomenon is more effective at improving reconstruction quality the wider the field-of-view. Note that the universality and efficiency of the spread spectrum technique was recently demonstrated on purely theoretical grounds beyond its application to radio interferometry [8]. The wide-field interferometric imaging framework thus provides intrinsic advantages, enhancing both sparsity and incoherence, and, consequently, the fidelity of reconstructed images.

## 3 Simulations

The wide-field interferometric imaging framework has been evaluated thoroughly on low-resolution simulated observations of sources with a Gaussian profile, where a direct comparison with planar reconstructions was made [6]. The predicted improvement in the fidelity of reconstructed images in the wide-field setting, due to the theoretical considerations discussed previously, is indeed realised in practice. A more realistic simulation of Galactic dust emission [9] at a higher resolution was also considered [6]. In Fig. 1 we plot the underlying spherical image of this simulation, and images reconstructed in the planar and spherical compressed sensing frameworks, with only 25% of Fourier samples measured (for comparison, the planar reconstruction is lifted to the sphere – the space where the image naturally lives). These images are recovered by solving the TV problem. The signal-to-noise-ratio (SNR) of the spherical reconstruction in the absence of the spread spectrum phenomenon is 7dB, while the reconstructed images in the presence of the spread spectrum phenomenon is 14dB and 19dB for the planar and spherical reconstructions respectively.

## 4 Conclusions and future

The intrinsic advantages of the $w$ component and spherical imaging for wide-field radio interferometry in the context of compressed sensing are clear. However, current techniques are necessarily somewhat idealise in order to remain as close as possible to the theoretical compressed sensing setting. Now that the effectiveness of these techniques has been demonstrated, it is of paramount importance to adapt them to realistic interferometric configurations. Furthermore, the possibility of optimising the configuration of interferometers to enhance the spread spectrum phenomenon for compressed sensing reconstruction is an exciting avenue of research at the level of acquisition. In summary, next-generation radio interferometric telescopes, such as the SKA, will inherently observe very large fields of view. Enhanced wide-field interferometric imaging techniques are therefore of increasing importance to ensure that the fidelity of reconstructed images keeps pace with the capabilities of new instruments.

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